CHAPTER 4

Quadratic Equations

1. OBJECTIVE QUESTIONS

1. If $\frac{1}{2}$ is a root of the equation $x^2 + kx - \frac{5}{4} = 0$, then the value of k is

(a) 2 (b) -2

(c) $\frac{1}{4}$ (d) $\frac{1}{2}$

Ans : (a) 2

Since, $\frac{1}{2}$ is a root of the quadratic equation

$$x^2 + kx - \frac{5}{4} = 0$$

Then,
$$\left(\frac{1}{2}\right)^2 + k\left(\frac{1}{2}\right) - \frac{5}{4} = 0$$
$$\frac{1}{4} + \frac{k}{2} - \frac{5}{4} = 0$$
$$\frac{1 + 2k - 5}{4} = 0$$
$$2k - 4 = 0$$
$$2k = 4$$
$$k = 2$$

2. Each root of $x^2 - bx + c = 0$ is decreased by 2. The resulting equation is $x^2 - 2x + 1 = 0$, then

(a) $b = 6, c = 9$ (b) $b = 3, c = 5$

(c) $b = 2, c = -1$ (d) $b = -4, c = 3$

Ans : (a) $b = 6, c = 9$

$$\alpha + \beta = b$$
$$\alpha\beta = c$$

According to the question

$$(\alpha + \beta - 4) = b - 4$$
$$(\alpha - 2)(\beta - 2) = \alpha\beta - 2(\alpha + \beta) + 4$$
$$= c - 2b + 4$$

Now
$$2 = b - 4$$
$$b = 6$$
$$1 = c - 2b + 4$$
$$1 = C - 2 \times 6 + 4$$
$$1 = C - 12 + 4$$
$$C = 1 + 12 - 4 = 9$$

3. Value (s) of k for which the quadratic equation $2x^2 - kx + k = 0$ has equal roots is/are

(a) 0 (b) 4

(c) 8 (d) 0, 8

Ans : (d) 0, 8

Given equation is, $2x^2 - kx + k = 0$

On comparing with $ax^2 + bx + c = 0$,

we get $a = 2,\ b = -k$ and $c = k$

For equal roots, the discriminant must be zero.

$$D = b^2 - 4ac = 0$$
$$(-k)^2 = -4(2)k = 0$$
$$k^2 - 8k = 0$$
$$k(k - 8) = 0$$
$$k = 0, 8$$

Hence, the required values of k are 0 and 8.

4. If the equation $(m^2 + n^2)x^2 - 2(mp + nq)x + p^2 + q^2 = 0$ has equal roots, then

(a) $mp = nq$ (b) $mq = np$

(c) $mn = pq$ (d) $mq = \sqrt{np}$

Ans : (b) $mq = np$

$$b^2 = 4ac'$$
$$4(mp + nq)^2 = 4(m^2 + n^2)(p^2 + q^2)$$
$$m^2q^2 + n^2p^2 - 2mnpq = 0$$
$$(mq - np)^2 = 0$$
$$mq - np = 0$$
$$mq = np$$

5. Which constant must be added and subtracted to solve the quadratic equation $9x^2 + \frac{3}{4}x - \sqrt{2} = 0$ by the method of completing the square?

(a) $\frac{1}{8}$ (b) $\frac{1}{64}$

(c) $\frac{1}{4}$ (d) $\frac{9}{64}$

Ans : (b) $\frac{1}{64}$

Given equation is $9x^2 + \frac{3}{4}x - \sqrt{2} = 0$

$$(3x)^2 + \frac{1}{4}(3x) - \sqrt{2} = 0$$

On putting $3x = y$,

We have,
$$y^2 + \frac{1}{4}y - \sqrt{2} = 0$$
$$y^2 + \frac{1}{4}y + \left(\frac{1}{8}\right)^2 - \left(\frac{1}{8}\right)^2 - \sqrt{2} = 0$$
$$\left(y + \frac{1}{8}\right)^2 = \frac{1}{64} + \sqrt{2}$$
$$\left(y + \frac{1}{8}\right)^2 = \frac{1 + 64 \cdot \sqrt{2}}{64}$$

Thus, $\frac{1}{64}$ must be added and subtracted to solve the given equation.

6. Any line is said to be a tangent to the curve, if it intersects the curve at one point. If the line $y = kx - 3$ is a tangent to the curve $y = 2x^2 + 7$, then the possible values of k is

(a) $4\sqrt{5}$ (b) $-4\sqrt{5}$

(c) Both (a) and (b) (d) None of these

Ans : (c) Both (a) and (b)

Given equations of line and curve are

$$y = kx - 3 \text{ and } y = 2x^2 + 7$$

Now, for point of intersection consider,

$$2x^2 + 7 = kx - 3$$

$$2x^2 - kx + 10 = 0$$

On comparing with $ax^2 + bx + c = 0$

we get $a = 2,\ b = -k$ and $c = 10$

Since, the lines is a tangent to the curve, so the discriminant $D = 0$.

i.e. $b^2 - 4ac = 0$

$$(-k)^2 - 4 \times 2 \times 10 = 0 \Rightarrow k^2 = 80$$

$$k = \pm 4\sqrt{5}$$

7. The linear factors of the quadratic equation $x^2 + kx + 1 = 0$ are

(a) $k \geq 2$ (b) $k \leq 2$

(c) $k \geq -2$ (d) $2 \leq k \leq -2$

Ans : (d) $2 \leq k \leq -2$

We have, $x^2 + kx + 1 = 0$

On comparing with $ax^2 + bx + c = 0$,

we get $a = 1,\ b = k$ and $c = 1$

For linear factors, $D \geq 0$

$$b^2 - 4ac \geq 0$$

$$k^2 - 4 \times 1 \times 1 \geq 0$$

$$(k^2 - 2^2) \geq 0$$

$$(k-2)(k+2) \geq 0$$

$$k \geq 2 \text{ and } k \leq -2$$

8. If the coefficient of x in the quadratic equation $x^2 + px + q = 0$ was taken as 17 in the place of 13 and its roots were found to be -2 and -15 then the roots of the original equation.

(a) 3,10 (b) $-3, -10$

(c) $-3, 10$ (d) $3, -10$

Ans : (b) $-3, -10$

Given, $x^2 + px + q = 0$

When we take the coefficient of x as 17, i.e. $p = 17$, then the roots are -2 and -15.

Thus, we can say -2 is a root of the equation

$$x^2 + 17x + q = 0$$

$$(-2)^2 + 17 \times (-2) + q = 0$$

$$4 - 34 + q = 0$$

$$q = 30$$

Clearly, the new quadratic equation will be

$$x^2 + 13x + 30 = 0$$

$$x^2 + 10x + 3x + 30 = 0$$

$$x(x+10) + 3(x+10) = 0$$

$$(x+10)(x+3) = 0$$

$$x + 10 = 0 \text{ or } x + 3 = 0$$

$$x = -10$$

or $x = -3$

9. If one root of the quadratic equation $ax^2 + bx + c = 0$ is the reciprocal of the other, then

(a) $b = c$ (b) $a = b$

(c) $ac = 1$ (d) $a = c$

Ans : (d) $a = c$

If one root is α, then the other $\frac{1}{\alpha}$.

$$\alpha \cdot \frac{1}{\alpha} = \text{product of roots} = \frac{c}{a}$$

$$1 = \frac{c}{a}$$

$$a = c$$

10. One of the two students, while solving a quadratic equation in x, copied the constant term incorrectly and got the roots 3 and 2. The other copied the constant term and coefficient of x^2 correctly as -6 and 1 respectively. The correct roots are

(a) $3, -2$ (b) $-3, 2$

(c) $-6, -1$ (d) $6, -1$

Ans : (d) $6, -1$

Let α, β be the roots of the equation.

Then, $\alpha + \beta = 5$

and $\alpha\beta = -6$.

So, the equation is

$$x^2 - 5x - 6 = 0$$

The roots of the equation are 6 and -1.

11. The quadratic equation $2x^2 - \sqrt{5}\,x + 1 = 0$ has

(a) two distinct real roots

(b) two equal real roots

(c) no real roots

(d) more than 2 real roots

Ans : (c) no real roots

Given equation is, $2x^2 - \sqrt{5x} + 1 = 0$

On comparing with $ax^2 + bx + c = 0$,

we get $a = 2,\ b = -\sqrt{5}$ and $c = 1$

Discriminant, $D = b^2 - 4ac$

$= (-\sqrt{5})^2 - 4 \times (2) \times (1)$

$= 5 - 8 = -3 < 0$

Since, discriminant is negative, therefore quadratic equation $2x^2 - \sqrt{5}x + 1 = 0$ has no real roots i.e., imaginary roots.

12. The real roots of the equation $x^{2/3} + x^{1/3} - 2 = 0$ are

(a) 1, 8 (b) $-1, -8$

(c) $-1, 8$ (d) $1, -8$

Ans : (d) $1, -8$

The given equation is

$$x^{2/3} + x^{1/3} - 2 = 0$$

Put $x^{1/3} = y$,

then $y^2 + y - 2 = 0$

$$(y-1)(y+2) = 0$$

$$y = 1$$

or $y = -2$

$$x^{1/3} = 1$$

or $x^{1/3} = -2$

$$x = (1)^3$$

or $x = (-2)^3 = -8$

Hence, the real roots of the given equations are 1, -8.

13. $(x^2+1)^2 - x^2 = 0$ has

(a) four real roots (b) two real roots

(c) no real roots (d) one real root

Ans : (c) no real roots

Given equation is,

$$(x^2+1)^2 - x^2 = 0$$

$$x^4 + 1 + 2x^2 - x^2 = 0 \quad [(a+b)^2 = a^2 + b^2 + 2ab]$$

$$x^4 + x^2 + 1 = 0$$

Let, $x^2 = y$

$$(x^2)^2 + x^2 + 1 = 0$$

$$y^2 + y + 1 = 0$$

On comparing with $ay^2 + by + c = 0$,

we get $a = 1,\ b = 1$ and $c = 1$

Discriminant, $D = b^2 - 4ac$

$= (1)^2 - 4(1)(1)$

$= 1 - 4 = -3$

Since, $D < 0$

$$y^2 + y + 1 = 0$$

i.e., $x^4 + x^2 + 1 = 0$

or $(x^2+1)^2 - x^2 = 0$ has no real roots.

14. The equation $2x^2 + 2(p+1)x + p = 0$, where p is real, always has roots that are

(a) Equal

(b) Equal in magnitude but opposite in sign

(c) Irrational

(d) Real

Ans : (d) Real

The discrimination of a quadratic equation $ax^2 + bx + c = 0$ is given by $b^2 - 4ac$.

Here, $a = 2,\ b = 2(p+1)$

and $c = p$

$b^2 - 4ac = [2(p+1)]^2 - 4(2p)$

$= 4(p+1)^2 - 8p$

$= 4[(p+1)^2 - 2p]$

$= 4[(p^2 + 2p + 1) - 2p]$

$= 4(p^2 + 1)$

For any real value of $p, 4(p^2+1)$ will always be positive as p^2 cannot be negative for real p.

Hence, the discriminant $b^2 - 4ac$ will always be positive When the discriminant is greater than '0' or is positive, then the roots of a quadratic equation will be real.

15. Out of a certain number of saras birds, one-fourth the number are moving about lotus plants, $\frac{1}{9}^{th}$ are coupled with $\frac{1}{4}^{th}$ as well as 7 times the square root of the number move on a hill, 56 birds remain in vakula tree. What is the total number of birds?

(a) 576 (b) 567

(c) 556 (d) 557

Ans : (a) 576

Let the total number of birds be x. Then, number of birds moving about lotus plants $= \frac{x}{4}$ and number of birds moving on a hill $= \frac{x}{9} + \frac{x}{4} + 7\sqrt{x}$.

Given,number of birds in vakula tree = 56

According to the given condition,

$$\frac{x}{4} + \left(\frac{x}{9} + \frac{x}{4} + 7\sqrt{x}\right) + 56 = x$$

$$x - \frac{x}{4} - \frac{x}{9} - \frac{x}{4} - 7\sqrt{x} - 56 = 0$$

$$\frac{36x - 9x - 4x - 9x}{36} - 7\sqrt{x} - 56 = 0$$

$$\frac{14x}{36} - 7\sqrt{x} - 56 = 0$$

$$\frac{7x}{18} - 7\sqrt{x} - 56 = 0$$

$$\frac{x}{18} - \sqrt{x} - 8 = 0$$

[dividing both sides by7]

$$x - 18\sqrt{x} - 144 = 0$$

Put $\sqrt{x} = y$, then above equation becomes

$$y^2 - 18y - 144 = 0$$

$$y^2 - 24y + 6y - 144 = 0$$

$$y(y-24) + 6(y-24) = 0$$

$$(y-24)(y+6) = 0$$

$\Rightarrow y = 24$ or -6

But $y \neq -6$

as $\sqrt{x} = y$

$$y = 24$$

$\Rightarrow \sqrt{x} = 24 \Rightarrow x = 576$

Hence, total number of birds is 576.

16. If $\sqrt{x+10} - \frac{6}{\sqrt{x+10}} = 5$, then extraneous root of this equation is

(a) 26 (b) -9

(c) -26 (d) 9

Ans : (b) -9

Given,

$$\sqrt{x+10} - \frac{6}{\sqrt{x+10}} = 5 \quad ...(1)$$

$$\frac{x+10-6}{\sqrt{x+10}} = 5$$

$$x+4 = 5(\sqrt{x+10})$$

On squaring both sides, we get

$$x^2+16+8x = 25(x+10)$$

$$x^2+8x-25x-250+16 = 0$$

$$x^2-17x-234 = 0$$

$$x^2-26x+9x-234 = 0$$

$$x(x-26)+9(x-26) = 0$$

$$(x+9)(x-26) = 0$$

$$x+9 = 0 \text{ or } x-26 = 0$$

$$x = 26 \text{ or } -9$$

On putting $x=-9$ in Eq. (1), we get

$$\sqrt{-9+10} - \frac{6}{\sqrt{-9+10}} = 5$$

$$1-\frac{6}{1} = 5 \Rightarrow -5 = 5$$

Which is not true.

Hence, extraneous root of given equation is -9.

17. If $\sin\alpha$ and $\cos\alpha$ are the roots of the equation $ax^2+bx+c=0$, then b^2 is

(a) c^2+2ac (b) a^2+ac

(c) a^2+2ac (d) c^2+ac

Ans : (c) a^2+2ac

Given equation is, $ax^2+bx+c = 0$

Since, $\sin\alpha$ and $\cos\alpha$ are the roots of the equation.

Sum of the roots, $\sin\alpha+\cos\alpha = \frac{-b}{a}$ ...(1)

and product of the roots, $\sin\alpha\cos\alpha = \frac{c}{a}$ ...(2)

On squaring both sides of Eq. (1), we get

$$(\sin\alpha+\cos\alpha)^2 = \left(\frac{-b}{a}\right)^2$$

$$\sin^2\alpha+\cos^2\alpha+2\sin\alpha\cos\alpha = \frac{b^2}{a^2}$$

$$1+2\sin\alpha\cos\alpha = \frac{b^2}{a^2}$$

$[\sin^2\theta+\cos^2\theta = 1]$

$$2\sin\alpha\cos\alpha = \frac{b^2}{a^2}-1$$

$$2\times\left(\frac{c}{a}\right) = \frac{b^2-a^2}{a^2}$$

[From Eq. 2]

$$2ac = b^2-a^2$$

$$b^2 = a^2+2ac$$

Hence proved.

18. Draw the graph of $y = x^2+x-12$. If $y=0$, then area of the triangle formed by joining the intersection point of curve.

(a) 12 sq. units (b) 24 sq. units

(c) 42 sq. units (d) 48 sq. units

Ans : (c) 42 sq. units

Given equation of curve is $y = x^2+x-12$

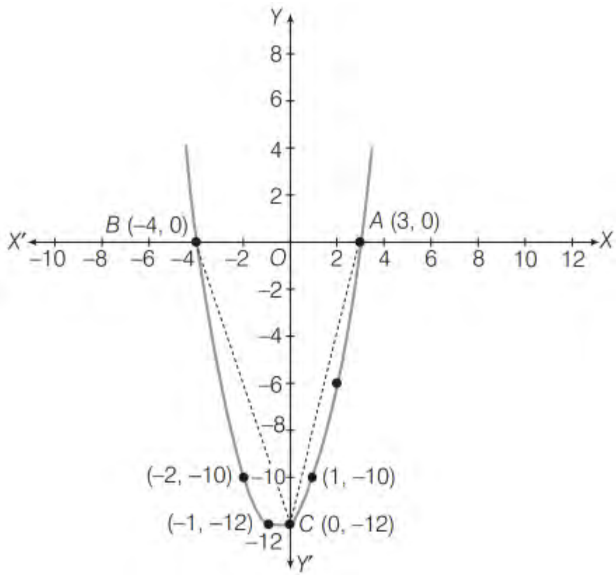
On comparing with $y = ax^2+bx+c$,

we get $a = 1 > 0$

So, it opens upwards

To draw its graph, we need some different values of y corresponding to different values of x.

x	-4	-2	-1	0	1	2	3
y	0	-10	-12	-12	-10	-6	0



We know that, abscissa of point of intersection of curve and X-axis, is the root of equation $y=0$. Here, we see that graph intersects the X-axis at two points A and C.

The roots of the given curve are 3 and -4.

Now, length of $AB = 4+3 = 7$

and, length of $OC = 12$

$$\text{Area of } \Delta ABC = \frac{1}{2}\times AB\times OC$$

$$= \frac{1}{2}\times 7\times 12 = 42 \text{ sq. units.}$$

19. Plot the roots of the equations $x^2-4x+3=0$ and $2y^2-7y+3=0$ and find the area of the smallest triangle formed by joining these points and origin.

(a) 0.5 sq units (b) 0.05 sq units

(c) 0.15 sq. units (d) 0.25 sq. units

Ans : (d) 0.25 sq. units

Given equation is, $x^2-4x+3 = 0$

$$x^2-3x-x+3 = 0 \quad \text{[by factorisation]}$$

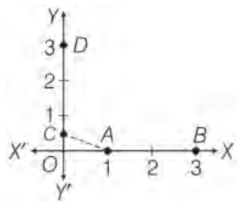
$$x(x-3)-1(x-3) = 0$$

$$(x-1)(x-3) = 0$$
$$x = 1 \text{ or } 3$$

Also, another given equation is,

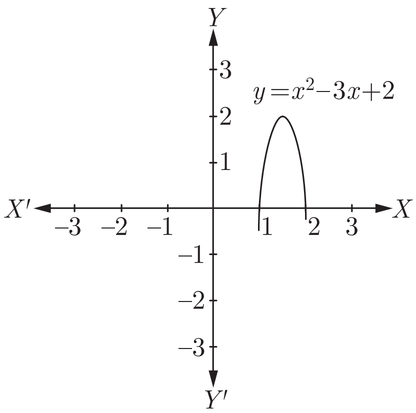
$$2y^2 - 7y + 3 = 0$$
$$2y^2 - 6y - y + 3 = 0$$
$$2y(y-3) - 1(y-3) = 0$$
$$(2y-1)(y-3) = 0$$
$$y = \frac{1}{2} \text{ or } 3$$

Now, let us plot these roots on the axes, which are shown below. The smallest triangle formed by joining these points and origin is OAC.



$$\text{Area of } \Delta\, OAC = \frac{1}{2} \times OC \times OA$$
$$= \frac{1}{2} \times \frac{1}{2} \times 1 = \frac{1}{4} \text{ sq. unit.}$$

20. A graph of quadratic polynomial is given below



If we rotate the axes at an angle of 90° in anti-clockwise direction, the figure remains at the same position. Find the equation of the graph.

(a) $y^3 + 3y + 2$ (b) $y^2 - 3y + 2$
(c) $y^2 + 2y + 3$ (d) $y^2 - 2y + 3$

Ans : (a) $y^3 + 3y + 2$

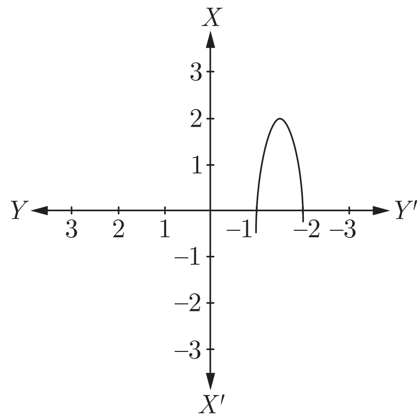
1. Given, $y = x^2 - 3x + 2$ and $y = 0$

$$x^3 - 3x + 2 = 0$$
$$x^2 - 2x - x + 2 = 0$$
$$x(x-2) - 1(x-2) = 0$$
$$(x-2)(x-1) = 0$$
$$x = 2$$

or $x = 1$

Hence, roots are 1 and 2.

2. When we rotate the axes at an angle of 90° in anti-clockwise direction, then the new graph is same as shown alongside.



Here, we see that graph is shown on negative of $Y-$ axis. So, we replace y by x and x by $-y$ in the original equation

$$y = x^2 - 3x + 2$$

Now, we get, $x = (-y)^2 - 3(-y) + 2$

$$x = y^2 + 3y + 2$$

21. The condition for one root of the quadratic equation $ax^2 + bx + c = 0$ to be twice the other, is

(a) $b^2 = 4ac$ (b) $2b^2 = 9ac$
(c) $c^2 = 4a + b^2$ (d) $c^2 = 9a - b^2$

Ans : (b) $2b^2 = 9ac$

$$\alpha + 2\alpha = -\frac{b}{a}$$

and $\alpha \times 2\alpha = \frac{c}{a}$

$$3\alpha = -\frac{b}{a}$$
$$\alpha = -\frac{b}{3a}$$

and $2\alpha^2 = \frac{c}{a}$

$$2\left(\frac{-b}{3a}\right)^2 = \frac{c}{a}$$
$$\frac{2b^2}{9a^2} = \frac{c}{a}$$
$$2ab^2 - 9a^2c = 0$$
$$a(2b^2 - 9ac) = 0$$

Since, $a \neq 0$

$$2b^2 = 9ac$$

Hence, the required condition is $2b^2 = 9ac$.

22. If $x^2 + y^2 = 25$, $xy = 12$, then $x =$

(a) $\{3, 4\}$ (b) $\{3, -3\}$
(c) $\{3, 4, -3, 4\}$ (d) $\{3, -3\}$

Ans : (c) $\{3, 4, -3, 4\}$

$$x^2 + y^2 = 25$$
$$xy = 12$$
$$x^2 + \left(\frac{12}{x}\right)^2 = 25$$
$$x^4 + 144 - 25x^2 = 0$$

$$(x^2-16)(x^2-9) = 0$$

Hence, $x^2 = 16$

and $x^2 = 9$

$x = \pm 4$

and $x = \pm 3$

23. If $x = \sqrt{7+4\sqrt{3}}$, then $x + \frac{1}{x} =$

(a) 4 (b) 6

(c) 3 (d) 2

Ans : (a) 4

We have $x = \sqrt{7+4\sqrt{3}}$

$$\frac{1}{x} = \frac{1}{\sqrt{7+4\sqrt{3}}}$$

$$= \frac{\sqrt{7-4\sqrt{3}}}{\sqrt{7+4\sqrt{3}} \cdot \sqrt{7-4\sqrt{3}}}$$

$$= \sqrt{7-4\sqrt{3}}$$

$$x + \frac{1}{x} = \sqrt{7+4\sqrt{3}} + \sqrt{7-4\sqrt{3}}$$

$$= (\sqrt{3}+2) + (2-\sqrt{3}) = 4$$

24. If the roots of the equation $px^2 + 2qx + r = 0$ and $qx^2 - 2\sqrt{prx} + q = 0$ be real, then

(a) $p = q$ (b) $q^2 = pr$

(c) $p^2 = qr$ (d) $r^2 = pq$

Ans : (b) $q^2 = pr$

Equation $px^2 + 2qx + r = 0$

and $qx^2 - 2\sqrt{pr} + x + q = 0$

have real roots then from first

$$4q^2 - 4pr \geq 0$$

$$q^2 \geq pr \quad \text{...(i)}$$

and from second $4(pr) - 4q^2 \geq 0$ (for real root)

$$pr \geq q^2 \quad \text{...(ii)}$$

From (i) and (ii), we get result

$$q^2 = pr$$

25. If the ratio of the roots of the equation $x^2 + bx + c = 0$ is the same as that of $x^2 + qx + r = 0$, then

(a) $r^2 b = qc^2$ (b) $r^2 c = qb^2$

(c) $c^2 r = q^2 b$ (d) $b^2 r = q^2 c$

Ans : (d) $b^2 r = q^2 c$

Let 1, 2 be the roots of equations (i) and (ii), 4 be the roots of equation (ii).

equations are $x^2 + 3x + 2 = 0$

and $x^2 - 6x + 8 = 0$

Comparing with $x^2 + bx + c = 0$

and $x^2 + qx + r = 0$

we get $b = -3 \quad c = 2$

$q = -6$

and $r = 8$.

Putting these values in the options, we find that option (d) is satisfied.

2. FILL IN THE BLANK

1. If the discriminant of a quadratic equation is zero, then its roots are .... and ....

Ans : real, equal

2. A polynomial of degree 2 is called the .... polynomial.

Ans : quadratic

3. If a, b are the roots of $x^2 + x + 1 = 0$, then $a^2 + b^2 =$ ....

Ans : -1

4. If α, β are the roots of $x^2 + bx + c = 0$ and $\alpha + h, \beta + h$ are the roots of $x^2 + qx + r = 0$, then $h =$ ....

Ans : $\frac{1}{2}(b-q)$

5. A quadratic equation cannot have more than .... roots.

Ans : two

6. Let $ax^2 + bx + c = 0$, where a, b, c are real numbers, $a \neq 0$, be a quadratic equation, then this equation has no real roots if and only if ....

Ans : $b^2 < 4ac$

7. The altitude of a right triangle is 7 cm less than its base. If the hypotenuse is 13 cm, the other two sides are ....

Ans : 5 cm, 12 cm

8. If the product ac in the quadratic equation $ax^2 + bx + c$ is negative, then the equation cannot have .... roots.

Ans : Non-real

9. The equation $ax^2 + bx + c = 0, a \neq 0$ has no real roots, if ....

Ans : $b^2 < 4ac$

10. A real number α is said to be .... of the quadratic equation $ax^2 + bx + c = 0$, if $a\alpha^2 + b\alpha + c = 0$.

Ans : root

11. The equation of the form $ax^2 + bx = 0$ will always have .... roots.

Ans : real

12. If the discriminant of a quadratic equation is greater than zero, then its roots are .... and ....

Ans : real, distinct

13. A quadratic equation in the variable x is of the form

$$ax^2 + bx + c = 0,$$

where a, b, c are real numbers and a ....

Ans : $\neq 0$

14. The roots of a quadratic equation is same as the .... of the corresponding quadratic polynomial.

Ans : zero

15. A quadratic equation $ax^2 + bx + c = 0$ has two distinct real roots, if $b^2 - 4ac$ ....
Ans : > 0

16. For any quadratic equation $ax^2 + bx + c = 0$, $b^2 - 4ac$, is called the .... of the equation.
Ans : discriminant

17. The values of k for which the equation $2x^2 + kx + x + 8 = 0$ will have real and equal roots are ....
Ans : 7 and – 9

18. A quadratic equation does not have any real roots if the value of its discriminant is .... zero.
Ans : less than

19. If α, β are roots of the equation $ax^2 + bx + c = 0$, then the quadratic equation whose roots are $a\alpha + b$ and $a\beta + b$ is ....
Ans : $x^2 - bx + ca = 0$

20. If r, s are roots of $ax^2 + bx + c = 0$, then $\frac{1}{r^2} + \frac{1}{s^2}$ is ....
Ans : $\frac{b^2 - 2ac}{c^2}$

21. The quadratic equation whose roots are the sum and difference of the squares of roots of the equation $x^2 - 3x + 2 = 0$ is ....
Ans : $x^2 - 8x + 15 = 0$

22. The equation $x^2 + x - 5 = 0$ then, product of its two roots is ....
Ans : -5

3. TRUE/FALSE

1. Sum of the reciprocals of the roots of the equation $x^2 + px + q = 0$ is $1/p$.
Ans : False

2. If the coefficient of x^2 and the constant term have the same sign and if the coefficient of x term is zero, then the quadratic equation has no real roots.
Ans : True

3. The nature of roots of equation $x^2 + 2x\sqrt{3} + 3 = 0$ are real and equal.
Ans : True

4. Every quadratic equation has at least one real root.
Ans : False

5. For the expression $ax^2 + 7x + 2$ to be quadratic, the possible values of a are non-zero real numbers.
Ans : True

6. Every quadratic equation has exactly one root.
Ans : False

7. A quadratic equation cannot be solved by the method of completing the square.
Ans : False

8. If the value of discriminant is equal to zero, then the equation has real and distinct roots.
Ans : False

9. 0.2 is a root of the equation $x^2 - 0.4 = 0$.
Ans : False

10. A quadratic equation has its degree at least two.
Ans : False

11. $(x^2 + 3x + 1) = (x - 2)^2$ is not a quadratic equation.
Ans : True

12. If the coefficient of x^2 and the constant term of a quadratic equation have opposite signs, then the quadratic equation has real roots.
Ans : True

13. $x^2 + x - 306 = 0$ represent quadratic equation where product of two consecutive positive integer is 306.
Ans : True

14. If we can factorise $ax^2 + bx + c, a \neq 0$, into a product of two linear factors, then the roots of the quadratic equation $ax^2 + bx + c = 0$ can be found by equating each factor to zero.
Ans : True

15. The equation $(x + 2)^2 = 0$ has real roots.
Ans : True

16. Every quadratic equation has at most two roots.
Ans : True

17. $(x - 2)(x + 1) = (x - 1)(x + 3)$ is a quadratic equation.
Ans : False

18. Every quadratic equation has at least two roots.
Ans : False

19. The roots of the equation $(x - 3)^2 = 3$ are $3 \pm \sqrt{3}$
Ans : True

20. The degree of a quadratic polynomial is atmost 2.
Ans : False

21. A quadratic equation may have no real root.
Ans : True

22. If sum of the roots is 2 and product is 5, then the quadratic equation is $x^2 - 2x + 5 = 0$
Ans : True

23. If 2 is a zero of the quadratic polynomial $p(x)$ then 2 is a root of the quadratic equation $p(x) = 0$.

Ans : True

24. If the product ac in the quadratic equation $ax^2 + bx + c$ is negative, then the equation cannot have non-real roots.

Ans : Ture

4. MATCHING QUESTIONS

DIRECTION : Each questions contains statements given in two coloumns which have to be matched. Statements (A, B, C, D) in coloumn-I have to be matched with statements (p, q, r, s) in column-II.

1. Column-II give roots of quadratic equations given in Column-I.

	Column-I		Column-II
(A)	$6x^2 + x - 12 = 0$	(p)	$(-6, 4)$
(B)	$8x^2 + 16x + 10 = 202$	(q)	$(9, 36)$
(C)	$x^2 - 45x + 324 = 0$	(r)	$(3, -1/2)$
(D)	$2x^2 - 5x - 3 = 0$	(s)	$(-3/2, 4/3)$

Ans : (A) – s, (B) – p, (C) – q, (D) – r.

1.
$$\begin{aligned} 6x^2 + x - 12 &= 0 \\ 6x^2 + 9x - 8x - 12 &= 0 \\ 3x(2x+3) - 4(2x+3) &= 0 \\ (3x-4)(2x+3x) &= 0 \\ x &= \frac{4}{3}, \frac{-3}{2} \end{aligned}$$

2.
$$\begin{aligned} 8x^2 + 16x - 192 &= 0 \\ 8x^2 + 48x - 32x - 192 &= 0 \\ 8x(x+6) - 32(x+6) &= 0 \\ x &= 4, 6 \end{aligned}$$

3.
$$\begin{aligned} x^2 - 45x + 324 &= 0 \\ x^2 - 36x - 9x + 324 &= 0 \\ x(x-36) - 9(x-36) &= 0 \\ x &= 9, 36 \end{aligned}$$

4.
$$\begin{aligned} 2x^2 - 5x - 3 &= 0 \\ 2x^2 - 6x + x - 3 &= 0 \\ 2x(x-3) + 1(x-3) &= 0 \\ x &= \frac{-1}{2}, 3 \end{aligned}$$

2.

	Column-I		Column-II
(A)	$(x-3)(x+4) + 1 = 0$	(p)	Forth degree polynomial
(B)	$(x+2)^3 = 2x(x^2-1)$	(q)	Quadratic equation
(C)	$(2x-2)^2 = 4x^2$	(r)	Non-quadratic equation
(D)	$(2x^2-2)^2 = 3$	(s)	Linear equation

Ans : (A) – q, (B) – r, (C) – s, (D) – p.

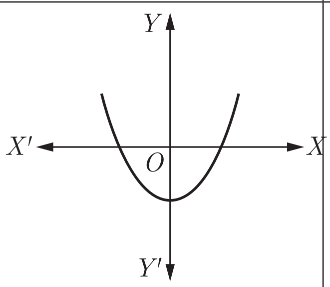
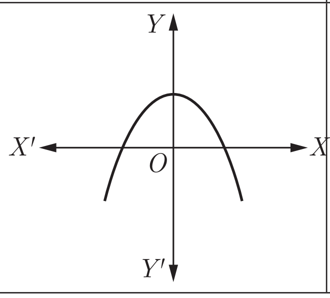
DIRECTION : Following questions has four statements (A, B, C and D) given in Column-I and four statements (p, q, r, s...) in Column-II. Any given statement in Coloumn-I can have correct matching with one or more statement (s) given in Column-II.

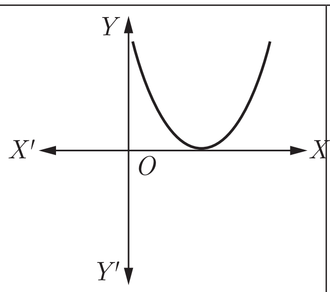
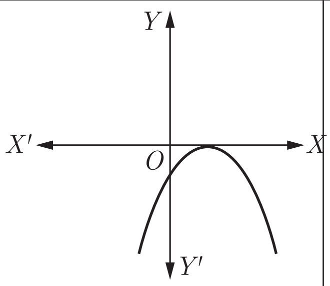
3.

	Column-I		Column-II
(A)	If α, β are roots of $ax^2 + bx + c = 0$ then one of the equation $ax^2 + bx(x-1) + c(x-1)^2 = 0$	(p)	$a < 0, b > 0$
(B)	If the roots of $ax^2 + b = 0$ are real, then	(q)	real and equal
(C)	Roots of $4x^2 - 4x + 1 = 0$	(r)	$\frac{\beta}{1+\beta}$
(D)	Roots of $(x-a)(x-b) + (x-b)(x-c) + (x-c)(x-a) = 0$ are always	(s)	$a > 0, b < 0$
		(t)	real
		(u)	$\frac{\alpha}{1+\alpha}$

Ans : (A) – (r, u), (B) – (p, s), (C) – q, (D) – t.

4. D be the discriminant of the quadratic equation $ax^2 + bx + c = 0$

	Column-I		Column-II
(A)		(p)	$a < 0$
(B)		(q)	$a > 0$

(C)		(r)	$D < 0$
(D)		(s)	$D > 0$
		(t)	$D = 0$

Ans : (A) – (q, s), (B) – (p, s), (C) – (q, r), (D) – (p, t).

5. ASSERTION AND REASON

DIRECTION : In the following questions, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
(c) Assertion (A) is true but reason (R) is false.
(d) Assertion (A) is false but reason (R) is true.

1. **Assertion :** $4x^2 - 12x + 9 = 0$ has repeated roots.
Reason : The quadratic equation $ax^2 + bx + c = 0$ have repeated roots if discriminant $D > 0$.

Ans : (c) Assertion (A) is true but reason (R) is false.

Assertion $\quad 4x^2 - 12x + 9 = 0$

$$D = b^2 - 4ac$$
$$= (-12)^2 - 4(4)(9)$$
$$= 144 - 144 = 0$$

Roots are repeated.

2. **Assertion :** The equation $x^2 + 3x + 1 = (x-2)^2$ is a quadratic equation.
Reason : Any equation of the form $ax^2 + bx + c = 0$ where $a \neq 0$, is called a quadratic equation.

Ans : (d) Assertion (A) is false but reason (R) is true.

We have, $x^2 + 3x + 1 = (x-2)^2 = x^2 - 4x + 4$

$\Rightarrow \quad x^2 + 3x + 1 = x^2 - 4x + 4$

$\Rightarrow \quad 7x - 3 = 0,$

it is not of the form $ax^2 + 6x + c = 0$

So, A is incorrect but R is correct.

3. **Assertion :** $(2x-1)^2 - 4x^2 + 5 = 0$ is not a quadratic equation.
Reason : $x = 0, 3$ are the roots of the equation $2x^2 - 6x = 0$.

Ans : (b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

Assertion and Reason both are true statements. But Reason is not the correct explanation.

Assertion $\quad (2x-1)^2 - 4x^2 + 5 = 0$

$$-4x + 6 = 0$$

Reason $\quad 2x^2 - 6x = 0$

$$2x(x-3) = 0$$
$$x = 0$$

and $\quad x = 3$

4. **Assertion :** The values of x are $-\frac{a}{2}, a$ for a quadratic equation $2x^2 + ax - a^2 = 0$.
Reason : For quadratic equation $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Ans : (d) Assertion (A) is false but reason (R) is true.

$$2x^2 + ax - a^2 = 0$$
$$x = \frac{-a \pm \sqrt{a^2 + 8a^2}}{4}$$
$$= \frac{-a + 3a}{4} = \frac{2a}{4}, \frac{-4a}{4}$$
$$x = \frac{a}{2}, -a$$

So, A is incorrect but R is correct.

5. **Assertion :** The equation $8x^2 + 3kx + 2 = 0$ has equal roots then the value of k is $\pm\frac{8}{3}$.
Reason : The equation $ax^2 + bx + c = 0$ has equal roots if $D = b^2 - 4ac = 0$

Ans : (a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

$$8x^2 + 3kx + 2 = 0$$

Discriminant, $\quad D = b^2 - 4ac$

$$D = (3k)^2 - 4 \times 8 \times 2 = 9k^2 - 64$$

For equal roots, $\quad D = 0$

$$9k^2 - 64 = 0$$
$$9k^2 = 64$$
$$k^2 = \frac{64}{9}$$
$$k = \pm\frac{8}{3}$$

So, A and R both are correct and R explains A.

6. **Assertion :** The value of $k = 2$, if one root of the quadratic equation $6x^2 - x - k = 0$ is $\frac{2}{3}$
Reason: The quadratic equation $ax^2 + bx + c = 0, a \neq 0$ has two roots.

Ans : (b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

As one root is $\frac{2}{3}$ $\quad x = \frac{2}{3}$

$$6 \times \left(\frac{2}{3}\right)^2 - \frac{2}{3} - k = 0$$

$$6 \times \frac{4}{9} - \frac{2}{3} = k$$

$$k = \frac{8}{3} - \frac{2}{3} = \frac{6}{3} = 2$$

$$k = 2$$

So, both A and R are correct but R does not explain A.

7. Assertion : The roots of the quadratic equation $x^2 + 2x + 2 = 0$ are imaginary.
Reason : If discriminant $D = b^2 - 4ac < 0$ then the roots of quadratic equation $ax^2 + bx + c = 0$ are imaginary.

Ans : (a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

$$x^2 + 2x + 2 = 0$$

Discriminant, $\quad D = b^2 - 4ac$

$$= (2)^2 - 4 \times 1 \times 2$$

$$= 4 - 8 = - < 04$$

Roots are imaginary.
So, both A and R are correct and R explains A.

8. Assertion : If roots of the equation $x^2 - bx + c = 0$ are two consecutive integers, then $b^2 - 4c = 1$
Reason : If a, b, c are odd integer then the roots of the equation $4abc\,x^2 + (b^2 - 4ac)x - b = 0$ are real and distinct.

Ans : (b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

Assertion : Given equation

$$x^2 - bx + c = 0$$

Let α, β be two roots such that

$$|\alpha - \beta| = 1$$

$$(\alpha + \beta)^2 - 4\alpha\beta = 1$$

$$b^2 - 4c = 1$$

Reason : Given equation

$$4abc\,x^2 + (b^2 - 4ac)x - b = 0$$

$$D = (b^2 - 4ac)^2 + 16ab^2c$$

$$D = (b^2 - 4ac)^2 > 0$$

Hence roots are real and unequal.

9. Assertion : The equation $9x^2 + 3kx + 4 = 0$ has equal roots for $k = \pm 4$.
Reason : If discriminant 'D' of a quadratic equation is equal to zero then the roots of equation are real and equal.

Ans : (a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

Assertion $\quad 9x^2 + 3kx + 4 = 0$

$$D = b^2 - 4ac$$

$$= (3k)^2 - 4(9)(4)$$

$$= 9k^2 - 144$$

For equal roots $\quad D = 0$

$$9k^2 = 144$$

$$k = \pm \frac{12}{3}$$

$$k = \pm 4$$

10. Assertion : A quadratic equation $ax^2 + bx + c = 0$, has two distinct real roots, if $b^2 - 4ac > 0$.
Reason : A quadratic equation can never be solved by using method of completing the squares.

Ans : (c) Assertion (A) is true but reason (R) is false.

11. Assertion : Sum and product of roots of $2x^2 - 3x + 5 = 0$ are $\frac{3}{2}$ and $\frac{5}{2}$ respectively.
Reason : If α and β are the roots of $ax^2 + bx + c = 0$, $a \neq 0$, then sum of roots $= \alpha + \beta = -\frac{b}{a}$ and product of roots $= \alpha\beta = \frac{c}{a}$.

Ans : (a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

Assertion and Reason both are correct and Reason is correct explanation.

Assertion $\quad 2x^2 - 3x + 5 = 0$

$$\alpha + \beta = \frac{-b}{a}$$

$$= \frac{-(-3)}{2} = \frac{3}{2}$$

and $\quad \alpha\beta = \frac{c}{a} = \frac{5}{2}$