CHAPTER 8

Introduction to Trigonometry

1. OBJECTIVE QUESTIONS

1. If $x = p\sec\theta$ and $y = q\tan\theta$, then

(a) $x^2 - y^2 = p^2q^2$ (b) $x^2q^2 - y^2p^2 = pq$

(c) $x^2q^2 - y^2p^2 = \frac{1}{p^2q^2}$ (d) $x^2q^2 - y^2p^2 = p^2q^2$

Ans : (d) $x^2q^2 - y^2p^2 = p^2q^2$

We know, $\sec^2\theta - \tan^2\theta = 1$

and $\sec\theta = \frac{x}{p}$

$\tan\theta = \frac{y}{q}$

$x^2q^2 - y^2p^2 = p^2q^2$

2. If $b\tan\theta = a$, the value of $\frac{a\sin\theta - b\cos\theta}{a\sin\theta + b\cos\theta}$ is

(a) $\frac{a-b}{a^2+b^2}$ (b) $\frac{a+b}{a^2+b^2}$

(c) $\frac{a^2+b^2}{a^2-b^2}$ (d) $\frac{a^2-b^2}{a^2+b^2}$

Ans : (d) $\frac{a^2-b^2}{a^2+b^2}$

$\tan\theta = \frac{a}{b}$

$\frac{a\sin\theta - b\cos\theta}{a\sin\theta + b\cos\theta} = \frac{a\tan\theta - b}{a\tan\theta + b} = \frac{a^2-b^2}{a^2+b^2}$

3. The value of $\tan 1^\circ \tan 2^\circ \tan 3^\circ \ldots \tan 89^\circ$ is

(a) 0 (b) 1

(c) ∞ (d) None of these

Ans : (b) 1

Given, $\tan 1^\circ \tan 2^\circ \tan 3^\circ \ldots \tan 89^\circ$

$= \tan(90^\circ - 89^\circ)\tan(90^\circ - 88^\circ)$

$\tan(90^\circ - 87^\circ)\ldots\tan 87^\circ \tan 88^\circ \tan 89^\circ$

$= \cot 89^\circ \cot 88^\circ \cot 87^\circ \ldots \tan 87^\circ$

$\tan 88^\circ \tan 89^\circ$

$= (\cot 89^\circ \tan 89^\circ)(\cot 88^\circ \tan 88^\circ)$

$(\cot 87^\circ \tan 87^\circ)\ldots(\cot 44^\circ \tan 44^\circ)\tan 45^\circ$

$= 1 \times 1 \times 1 \ldots 1 \times 1 = 1$

4. $(\cos^4 A - \sin^4 A)$ is equal to

(a) $1 - 2\cos^2 A$ (b) $2\sin^2 A - 1$

(c) $\sin^2 A - \cos^2 A$ (d) $2\cos^2 A - 1$

Ans : (d) $2\cos^2 A - 1$

$(\cos^4 A - \sin^4 A) = (\cos^2 A)^2 - (\sin^2 A)^2$

$= (\cos^2 A - \sin^2 A)(\cos^2 A + \sin^2 A)$

$= (\cos^2 A - \sin^2 A)(1)$

$= \cos^2 A - (1 - \cos^2 A)$

$= 2\cos^2 A - 1$

5. If $\sec 5A = \text{cosec}(A + 30^\circ)$, where $5A$ is an acute angle, then the value of A is

(a) 15° (b) 5°

(c) 20° (d) 10°

Ans : (d) 10°

We have, $\sec 5A = \text{cosec}(A + 30^\circ)$

$\sec 5A = \sec[90^\circ - (A - 30^\circ)]$

$[\sec(90^\circ - \theta) = \text{cosec}\,\theta]$

$\sec 5A = \sec(60^\circ - A)$

$5A = 60^\circ - A$

$6A = 60^\circ$

$A = 10^\circ$

6. If $x\sin^3\theta + y\cos^3\theta = \sin\theta\cos\theta$ and $x\sin\theta = y\cos\theta$, than $x^2 + y^2$ is equal to

(a) 0 (b) 1/2

(c) 1 (d) 3/2

Ans : (c) 1

We have, $x\sin^3\theta + y\cos^3\theta = \sin\theta\cos\theta$

$(x\sin\theta)\sin^2\theta + (y\cos\theta)\cos^2\theta = \sin\theta\cos\theta$

$x\sin\theta(\sin^2\theta) + (x\sin\theta)\cos^2\theta = \sin\theta\cos\theta$

$x\sin\theta(\sin^2\theta + \cos^2\theta) = \sin\theta\cos\theta$

$x\sin\theta = \sin\theta\cos\theta \Rightarrow x = \cos\theta$

Now, $x\sin\theta = y\cos\theta$

$\cos\theta\sin\theta = y\cos\theta$

$y = \sin\theta$

Hence, $x^2 + y^2 = \cos^2\theta + \sin^2\theta = 1$

7. If $\tan 2A = \cot(A - 18^\circ)$, where $2A$ is an acute angle, then the value of A is

(a) 12° (b) 18°

(c) 36° (d) 48°

Ans : (c) 36°

Given, $\tan 2A = \cot(A - 18^\circ)$

$\cot(90^\circ - 2A) = \cot(A - 18^\circ)$

$90^\circ - 2A = A - 18^\circ$

[since, $(90^\circ - 2A)$ and $(A - 18^\circ)$ both are acute angles]

$90^\circ + 18^\circ = A + 2A$

$3A = 108^\circ$

$A = \frac{108^\circ}{3} = 36^\circ$

8. If $\tan\theta + \sin\theta = m$ and $\tan\theta - \sin\theta = n$, then $m^2 - n^2$ is equal to

(a) $\sqrt{mn}$ (b) $\sqrt{\frac{m}{n}}$

(c) $4\sqrt{mn}$ (d) None of these

Ans : (c) $4\sqrt{mn}$

Given, $\tan\theta + \sin\theta = m$ and $\tan\theta - \sin\theta = n$

$m^2 - n^2 = (\tan\theta + \sin\theta)^2 - (\tan\theta - \sin\theta)^2$

$= 4\tan\theta\sin\theta$

$= 4\sqrt{\tan^2\theta\sin^2\theta}$

$= 4\sqrt{\sin^2\theta\frac{\sin^2\theta}{\cos^2\theta}}$

$= 4\sqrt{\frac{\sin^2\theta}{\cos^2\theta} - \sin^2\theta}$

$= 4\sqrt{\tan^2\theta - \sin^2\theta}$

$= 4\sqrt{(\tan\theta + \sin\theta)(\tan\theta - \sin\theta)}$

$= 4\sqrt{mn}$

9. If $0 < \theta < \frac{\pi}{4}$, then the simplest form of $\sqrt{1 - 2\sin\theta\cos\theta}$ is

(a) $\sin\theta - \cos\theta$ (b) $\cos\theta - \sin\theta$

(c) $\cos\theta + \sin\theta$ (d) $\sin\theta\cos\theta$

Ans : (b) $\cos\theta - \sin\theta$

$\sqrt{1 - 2\sin\theta\cos\theta} = \sqrt{\sin^2\theta + \cos^2\theta - 2\sin\theta\cos\theta}$

$= \sqrt{(\cos\theta - \sin\theta)^2}$

$[a^2 + b^2 - 2ab = (a - b)^2$ and $1 = \sin^2\theta + \cos^2\theta]$

$= \cos\theta - \sin\theta$

For $0^\circ < \theta < 45^\circ$

	0	$\pi/6$	$\pi/4$
$\cos\theta$	1	$\sqrt{3}/2$	$1/\sqrt{2}$
$\sin\theta$	0	$1/2$	$1/\sqrt{2}$

Here, we see that $\cos\theta > \sin\theta$, when $0 < \theta < \pi/4$, that's why we take $(\cos\theta - \sin\theta)^2$ instead of taking $(\sin\theta - \cos\theta)^2$.

10. If $(\sec A + \tan A)(\sec B + \tan B)(\sec C + \tan C) = (\sec A - \tan A)(\sec B - \tan B)(\sec C - \tan C) = x$ then the value/ values of x is/are

(a) ± 1 (b) 0

(c) ± 2 (d) 1

Ans : (a) ± 1

We have, $(\sec A + \tan A)(\sec B + \tan B)(\sec C + \tan C)$

$= (\sec A - \tan A)$

$(\sec B - \tan B)(\sec C - \tan C)$

On multiplying both sides by

$(\sec A - \tan A)(\sec B - \tan B)(\sec C - \tan C)$, we get

$(\sec A + \tan A)(\sec B + \tan B)(\sec C + \tan C)$

$\times (\sec A - \tan A)(\sec B - \tan B)(\sec C - \tan C)$

$= (\sec A - \tan A)^2(\sec B - \tan B)^2(\sec C - \tan C)^2$

$(\sec^2 A - \tan^2 A)(\sec^2 B - \tan^2 B)(\sec^2 C - \tan^2 C)$

$= (\sec A - \tan A)^2(\sec B - \tan B)^2(\sec C - \tan C)^2$

$[(a + b)(a - b) = a^2 - b^2]$

$1 \times 1 \times 1 = [(\sec A - \tan A)(\sec B - \tan B)(\sec C - \tan C)]^2$

$[\sec^2\theta - \tan^2\theta = 1]$

$(\sec A - \tan A)(\sec B - \tan B)(\sec C - \tan C) = \pm 1$

11. If $\sin\theta + \sin^2\theta = 1$, then find the value of $\cos^{12}\theta + 3\cos^{10}\theta + 3\cos^8\theta + \cos^6\theta + 2\cos^4\theta + 2\cos^2\theta - 2$.

(a) 0 (b) 1

(c) $\cos\theta$ (d) $\sin\theta$

Ans : (b) 1

We have, $\sin\theta + \sin^2\theta = 1$

$\sin\theta = 1 - \sin^2\theta$

$\sin\theta = \cos^2\theta$

$[\sin^2\theta + \cos^2\theta = 1]$

$\cos^{12}\theta + 3\cos^{10}\theta + 3\cos^8\theta + \cos^6\theta + 2\cos^4\theta + 2\cos^2\theta - 2$

$= (\cos^{12}\theta + 3\cos^{10}\theta + 3\cos^8\theta + \cos^6\theta)$

$+ 2(\cos^4\theta + \cos^2\theta - 1)$

$= (\cos^4\theta)^3 + 3\cos^6\theta(\cos^4\theta + \cos^2\theta)$

$+ (\cos^2\theta)^3 + 2(\cos^4\theta + \cos^2\theta - 1)$

$= (\cos^4\theta + \cos^2\theta)^3 + 2(\cos^4\theta + \cos^2\theta - 1)$

$[(a + b)^3 = a^3 + b^3 + 3ab(a + b)]$

$= (\sin^2\theta + \cos^2\theta)^3 + 2(\sin^2\theta + \cos^2\theta - 1)$

$[\cos^2\theta = \sin\theta \Rightarrow \cos^4\theta = \sin^2\theta]$

$= (1)^3 + 2(1 - 1) = 1$

$[\sin^2\theta + \cos^2\theta = 1]$

12. If $0^\circ < x < 90^\circ$ and $2\sin x + 15\cos^2 x = 7$, then find the value of $\tan x$.

(a) 4/5 (b) 3/5

(c) 3/4 (d) 4/3

Ans : (d) 4/3

Given, $2\sin x + 15\cos^2 x = 7$

$2\sin x + 15(1 - \sin^2 x) = 7$ $[\sin^2 x + \cos^2 x = 1]$

$2\sin x + 15 - 15\sin^2 x = 7$

$15\sin^2 x - 2\sin x - 8 = 0$

Let, $y = \sin x$, then

$$15y^2 - 2y - 8 = 0$$
$$15y^2 - 12y + 10y - 8 = 0$$
[by splitting the middle term]
$$3y(5y-4) + 2(5y-4) = 0$$
$$(5y-4)(3y+2) = 0$$
$$5y - 4 = 0$$
and $3y + 2 = 0 \Rightarrow y = 4/5$
and $y = -2/3$
$$\sin x = \frac{4}{5}$$
and $\sin x = -\frac{2}{3}$ [put $y = \sin x$]
But, $0^\circ < x < 90^\circ$
$$\sin x = 4/5$$
[for $0^\circ < x < 90^\circ$, $\sin x$ is not negative]
Now, $\cos x = \sqrt{1 - \sin^2 x}$
$$= \sqrt{1 - \left(\frac{4}{5}\right)^2}$$
$$= \sqrt{1 - \frac{16}{25}} = \sqrt{\frac{9}{25}}$$
$$= \frac{3}{5}$$
$$\tan x = \frac{\sin x}{\cos x} = \frac{4/5}{3/5} = \frac{4}{3}$$

13. If $f(x) = \cos^2 x + \sec^2 x$, then $f(x)$

(a) ≥ 1 (b) ≤ 1
(c) ≥ 2 (d) ≤ 2

Ans : (c) ≥ 2

Given, $f(x) = \cos^2 x + \sec^2 x$
$$= \cos^2 x + \sec^2 x - 2 + 2$$
[adding and subtracting 2]
$$= \cos^2 x + \sec^2 x - 2\cos x \cdot \sec x + 2$$
$[\cos x \cdot \sec x = 1]$
$$= (\cos x - \sec x)^2 + 2$$
$[a^2 + b^2 - 2ab = (a-b)^2]$
We know that, square of any expression is always greater than equal to zero.
$f(x) \geq 2$ Hence proved.

14. If ABC is a right angled triangle, then find the relation between
$\tan\left(\frac{A-B-C}{2}\right)$ and $-\tan\left(\frac{A+B-C}{2}\right)$

(a) equal (b) unequal
(c) sum of these equal to 1 (d) None of the above

Ans : (b) unequal
Given, ABC is a right angled triangle.
Since, the sum of the angles of a triangle is 180°.
$$A + B + C = 180^\circ$$
Now, $\tan\left(\frac{A-B-C}{2}\right) = \tan\left[\frac{A-(180^\circ - A)}{2}\right]$
[from Eq. (1)] $= \tan\left(\frac{2A - 180^\circ}{2}\right)$
$= \tan(A - 90^\circ)$
$= \tan[-(90^\circ - A)]$
$= -\tan(90^\circ - A)$
$[\tan(-\theta) = -\tan\theta]$
$= -\cot A$
$[\tan(90^\circ - A) = \cot A]$ ...(2)
and $-\tan\left(\frac{A+B-C}{2}\right) = -\tan\left[\frac{(180^\circ - C)C}{2}\right]$
[from Eq. (1)] $= -\tan(90^\circ - C)$
$= \cot C$ ...(3)
From. Eq. (2) and Eq. (3), we get
$$\tan\left(\frac{A-B-C}{2}\right) \neq -\tan\left(\frac{A+B-C}{2}\right)$$

15. If $\sin\theta + \sin^2\theta + \sin^3\theta = 1$, then $\cos^6\theta - 4\cos^4\theta + 8\cos^2\theta$ is equal to

(a) 1 (b) 2
(c) 3 (d) 4

Ans : (d) 4

Given, $\sin\theta + \sin^2\theta + \sin^3\theta = 1$
$$\sin\theta + \sin^3\theta = 1 - \sin^2\theta$$
$$\sin\theta(1 + \sin^2\theta) = \cos^2\theta$$
$$\sin\theta(1 + 1 - \cos^2\theta) = \cos^2\theta$$
$[\cos^2\theta + \sin^2\theta = 1]$
$$\sin\theta(2 - \cos^2\theta) = \cos^2\theta$$
On squaring both sides, we get,
$$\sin^2\theta(2 - \cos^2\theta)^2 = \cos^4\theta$$
$$(1 - \cos^2\theta)(4 + \cos^4\theta - 4\cos^2\theta) = \cos^4\theta$$
$$4 + \cos^4\theta - 4\cos^2\theta - 4\cos^2\theta - \cos^6\theta + 4\cos^4\theta = \cos^4\theta$$
$$4 = \cos^6\theta - 4\cos^4\theta - \cos^4\theta + 4\cos^2\theta + 4\cos^2\theta + \cos^4\theta$$
$$4 = \cos^6\theta - 4\cos^4\theta + 8\cos^2\theta$$

16. If $m = a\cos^3\theta + 3a\cos\theta\sin^2\theta$ and $n = a\sin^3\theta + 3a\cos^2\theta\sin\theta$, then $(m+n)^{2/3} + (m-n)^{2/3}$ is equal to

(a) $2a^{2/3}$ (b) $a^{2/3}$
(c) $2a^{3/2}$ (d) $a^{3/2}$

Ans : (a) $2a^{2/3}$

Given, $m = a\cos^3\theta + 3a\cos\theta\sin^2\theta$ ...(1)
and $n = a\sin^3\theta + 3a\cos^2\theta\sin\theta$ ...(2)
On adding Eqs. (1) and (2), we get
$$m + n = a\cos^3\theta + 3a\cos\theta\sin^2\theta + a\sin^3\theta + 3a\cos^2\theta\sin\theta$$
$$= a[\cos^3\theta + \sin^3\theta + 3\cos\theta\sin\theta(\sin\theta + \cos\theta)]$$
$$= a(\cos\theta + \sin\theta)^3 \quad (3)$$
$[(a+b)^3 = a^3 + b^3 + 3ab(a+b)]$
On subtracting Eq. (2) from Eq. (1), we get
$$m - n = a\cos^3\theta + 3a\cos\theta\sin^2\theta - a\sin^3\theta - 3a\cos^2\theta\sin\theta$$
$$= a[\cos^3\theta - \sin^3\theta - 3\cos\theta\sin\theta(\cos\theta - \sin\theta)]$$
$$= a[\cos\theta - \sin\theta]^3 \quad ...(4)$$
$[(a-b)^3 = a^3 - b^3 - 3ab(a-b)]$

Now, $(m+n)^{2/3}+(m-n)^{2/3} = \left[a(\cos\theta+\sin\theta)^3\right]^{2/3} + \left[a(\cos\theta-\sin\theta)^3\right]^{2/3}$ [from Eq. (3) and (4)]

$= a^{2/3}(\cos\theta+\sin\theta)^2 + a^{2/3}(\cos\theta-\sin\theta)^2$

$= a^{2/3}\left[(\cos\theta+\sin\theta)^2+(\cos\theta-\sin\theta)^2\right]$

$= a^{2/3}\left[\cos^2\theta+\sin^2\theta+2\cos\theta\sin\theta+\cos^2\theta+\sin^2\theta-2\sin\theta\cos\theta\right]$

$\left[(a+b)^2 = a^2+b^2+2ab, (a-b)^2 = a^2+b^2-2ab\right]$

$= a^{2/3}[1+1]$ $[\cos^2\theta+\sin^2\theta = 1]$

$= 2a^{2/3}$

17. If $\tan\theta = \dfrac{a\sin\phi}{1-a\cos\phi}$ and $\tan\phi = \dfrac{b\sin\theta}{1-b\cos\theta}$, then $\dfrac{a}{b} =$

(a) $\dfrac{\sin\theta}{1-\cos\theta}$ (b) $\dfrac{\sin\theta}{1-\cos\phi}$

(c) $\dfrac{\sin\phi}{\sin\theta}$ (d) $\dfrac{\sin\theta}{\sin\phi}$

Ans : (d) $\dfrac{\sin\theta}{\sin\phi}$

We have, $\tan\theta = \dfrac{a\sin\phi}{1-a\cos\phi}$

$\cot\theta = \dfrac{1}{a\sin\phi} - \cot\phi$

$\cot\theta+\cot\phi = \dfrac{1}{a\sin\phi}$ ...(1)

$\tan\phi = \dfrac{b\sin\theta}{1-b\cos\theta}$

$\cot\phi = \dfrac{1}{b\sin\theta} - \cot\theta$

$\cot\phi+\cot\theta = \dfrac{1}{b\sin\theta}$ ...(2)

From (1) and (2), we have

$\dfrac{1}{a\sin\phi} = \dfrac{1}{b\sin\theta}$

$\dfrac{a}{b} = \dfrac{\sin\theta}{\sin\phi}$

18. If $a\sec\theta+b\tan\theta+c=0$ and $p\sec\theta+q\tan\theta+r=0$, then $(br-qc)^2-(pc-ar)^2$ is equal to

(a) $(ap-bq)^2$ (b) $(aq-bp)^2$

(c) $(ap-bq)$ (d) $(aq-bp)$

Ans : (b) $(aq-bp)^2$

We have, $a\sec\theta+b\tan\theta+c = 0$

and $p\sec\theta+q\tan\theta+r = 0$

Solving these two equations for $\sec\theta$ and $\tan\theta$ by the cross-multiplication method, we get

$\dfrac{\sec\theta}{br-qc} = \dfrac{\tan\theta}{cp-ar} = \dfrac{1}{aq-bp}$

$\sec\theta = \dfrac{br-cq}{aq-bp}$

and $\tan\theta = \dfrac{cp-ar}{aq-bp}$

Now, $\sec^2\theta-\tan^2\theta = 1$

$\left(\dfrac{br-cq}{aq-bp}\right)^2-\left(\dfrac{cp-ar}{aq-bp}\right)^2 = 1$

$(br-cq)^2-(cp-ar)^2 = (aq-bp)^2$

2. FILL IN THE BLANK

1. $\sin 60^\circ\cos 30^\circ+\sin 30^\circ\cos 60^\circ =$ .....

Ans : 1

2. $\sin^2\theta+\sin^2(90^\circ-\theta) =$ .....

Ans : 1 [**Hint :** $\sin^2(90^\circ-\theta)] = \cos^2\theta$]

3. $2\tan^2 45^\circ+3\cos^2 30^\circ-\sin^2 60^\circ =$ .....

Ans : $\dfrac{7}{2}$

4. Triangle in which we study trigonometric ratios is called .....

Ans : Right Triangle

5. $\dfrac{\cos 45^\circ}{\sec 30^\circ+\operatorname{cosec} 30^\circ} =$ .....

Ans : $\dfrac{3(\sqrt{3}-1)}{4}$

6. $\dfrac{\sin 18^\circ}{\cos 72^\circ} =$ .....

Ans : 1

7. $\cos 48^\circ-\sin 42^\circ =$ .....

Ans : 0

8. Cosine of 90° is .....

Ans : Zero

9. If $15\cot A = 8$, $\sec A =$ .....

Ans : 17/8

10. The value of $\sin A$ or $\cos A$ never exceeds .....

Ans : 1

11. sine of $(90-\theta)$ is .....

Ans : $\cos\theta$

12. $\sin^2 A+\cos^2 A =$ .....

Ans : 1

13. It $\tan A = 4/3$ then $\sin A$ .....

Ans : 4/5

14. In a right triangle ABC, right angled at B, if $\tan A = 1$, $\sin A\cos A =$ .....

Ans : $\dfrac{1}{2}$

15. Reciprocal of $\sin\theta$ is .....

Ans : $\operatorname{cosec}\theta$

16. In ΔABC, right-angled at B, $AB = 24$ cm, $BC = 7$ cm . $\sin A =$ .....

Ans : 7/25

17. Maximum value for sine of any angle is .....
Ans : 1

18. In ΔPQR, right-angled at Q, $PR + QR = 25\,\text{cm}$ and $PQ = 5\,\text{cm}$. The value of $\tan P$ is .....
Ans : $12/5$

19. Sum of ..... of sine and cosine of angle is one.
Ans : Square

3. TRUE/FALSE

1. The value of $\sin\theta$ increases as θ increases.
Ans : True

2. $\sqrt{(1-\cos^2\theta)\sec^2\theta} = \tan\theta$
Ans : True

3. $\sec A = \frac{12}{5}$ for some value of angel A.
Ans : True

4. $\sin(A+B) = \sin A + \sin B$.
Ans : False

5. The value of $\cos\theta$ increases as θ increases.
Ans : False

6. $\sin\theta = \frac{5}{3}$ for some angle θ.
Ans : False

7. The value of $\tan A$ is always less than 1.
Ans : False

8. The value of the expression $(\cos^2 23^\circ - \sin^2 67^\circ)$ is positive.
Ans : False

9. $\cot A$ is not defined for $A = 0^\circ$.
Ans : True

10. $\sin(90^\circ - A) = \cos A$
Ans : True

11. If $\angle B$ and $\angle Q$ are acute angles such that $\sin B = \sin Q$, then $\angle B \neq \angle Q$.
Ans : False

12. $(\tan\theta + 2)(2\tan\theta + 1) = 5\tan\theta + \sec^2\theta$
Ans : False

13. $\frac{\tan 65^\circ}{\tan 25^\circ} = 1$
Ans : False

14. $\sin\theta = \cos\theta$ for all values of θ.
Ans : False

15. The value of the expression $(\sin 80^\circ - \cos 80^\circ)$ is negative
Ans : False

16. $\tan 48^\circ \tan 23^\circ \tan 42^\circ \tan 67^\circ \neq 1$
Ans : False

17. Trigonometry deals with measurement of components of triangles.
Ans : True

18. $\sin^2\theta \times \cos^2\theta = 1$
Ans : False

19. The value of $\tan A$ is always less than 1.
Ans : False

20. $\frac{\tan 47^\circ}{\cot 43^\circ} = 1$
Ans : True

21. $\sec A = 12/5$ for some value of angle A.
Ans : True

22. If $\cos A + \cos^2 A = 1$, then $\sin^2 A + \sin^4 A = 1$.
Ans : True

23. $\cos A$ is the abbreviation used for the cosecant of angle A.
Ans : False

24. $\cot A$ is the product of cot and A.
Ans : False

25. The value of $\sin\theta + \cos\theta$ is always greater than 1.
Ans : False

26. $\sin\theta = \frac{4}{3}$ for some angle θ.
Ans : False

4. MATCHING QUESTIONS

DIRECTION : Each question contains statements given in two Columns which have to be matched. Statements (A, B, C, D) in Column-I have to be matched with statements (p, q, r, s) in Column-II.

1. In ΔABC, $\angle B = 90^\circ$, $AB = 3\,\text{cm}$ and $BC = 4\,\text{cm}$ then match the column.

	Column-I		**Column-II**
(A)	$\sin C$	(p)	3/5
(B)	$\cos C$	(q)	4/5
(C)	$\tan A$	(r)	5/3
(D)	$\sec A$	(s)	4/3

Ans : (A) – p, (B) – q, (C) – s, (D) – r

2.

	Column-I		Column-II
(A)	$\frac{\cos A}{1+\sin A}+\frac{1+\sin A}{\cos A}$	(p)	$\operatorname{cosec} A+\cot A$
(B)	$\frac{\cos A-\sin A+1}{\cos A+\sin A-1}$	(q)	$2\sec A$
(C)	$\sqrt{\frac{1+\sin A}{1-\sin A}}$	(r)	$\sec A+\tan A$
(D)	$\frac{\sin^2 A}{1-\cos A}$	(s)	$\frac{1+\sec A}{\sec A}$

Ans : (A) – q, (B) – p, (C) – r, (D) – s

DIRECTION : Following question has four statements (A, B, C, and D) given in Column-I and six statements (p, q, r, s, t and u) in Colunn-II. Any given statement in Column-I can have correct matching with one or more statement(s) given in Column-II.

3. If $\sin A=\frac{7}{25}$, then

	Column-I		Column-II
(A)	$\cos A$	(p)	24/25
(B)	$\tan A$	(q)	7/24
(C)	$\operatorname{cosec} A$	(r)	25/7
(D)	$\sec A$	(s)	25/24
		(t)	$1-1/25$
		(u)	$1+1/24$

Ans : (A) – (p, t), (B) – q, (C) – r, (D) – (s, u)

DIRECTION : Match option of Column I with the appropriate option of Column II.

4.

Column I		Column II	
(A)	$\sin^2 37^\circ+\sin^2 53^\circ+\sin^2 90^\circ$	(p)	0
(B)	$\tan 35^\circ \tan 45^\circ \tan 55^\circ$	(q)	3
(C)	$\frac{\sec 72^\circ \sin 18^\circ+\tan 72^\circ \cot 18^\circ}{\cos 60^\circ}$	(r)	1
(D)	$\frac{\tan 60^\circ}{\tan 30^\circ}$	(s)	2
(E)	$\sin^2 30^\circ+\cos^2 30-\sin^2 60^\circ-\cos^2 60$	(t)	4

Ans :

(A) – s, (B) – r, (C) – t, (D) – q, (E) – p.

5. ASSERTION AND REASON

DIRECTION : In the following questions, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

1. **Assertion :** The value of $\sin\theta=\frac{4}{3}$ in not possible.
Reason : Hypotenuse is the largest side in any right angled triangle.

Ans : (a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

$$\sin\theta=\frac{P}{H}=\frac{4}{3}$$

Here, perpendicular is grater than the hypotenuse which is not possible in any right triangle.

2. **Assertion :** $\sin^2 67^\circ+\cos^2 67^\circ=1$
Reason : For any value of $\theta, \sin^2\theta+\cos^2\theta=1$

Ans : (a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

$$\sin^2\theta+\cos^2\theta=1$$
$$\sin^2 67^\circ+\cos^2 67^\circ=1$$

3. **Assertion :** The value of $\sec^2 10^\circ-\cot^2 80^\circ$ is 1
Reason : The value of $\sin 30^\circ=\frac{1}{2}$

Ans : (b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

We have,

$$\sec^2 10^\circ-\cot^2 80^\circ=\sec^2 10^\circ-\cot^2(90^\circ-10^\circ)$$
$$=\sec^2 10^\circ-\tan^2 10^\circ=1$$

Also, $\sin 30^\circ=\frac{1}{2}$

4. **Assertion :** $\sin 47^\circ=\cos 43^\circ$

Reason : $\sin\theta=\cos(90+\theta)$, where θ is an acute angle.

Ans : (c) Assertion (A) is true but reason (R) is false. Assertion is true, but reason is not correct.

$$\sin\theta=\cos(90-\theta)$$
$$\sin 47^\circ=\cos(90-47^\circ)=\cos 43^\circ$$

5. **Assertion :** In a right angled triangle, if $\tan\theta=\frac{3}{4}$, the greatest side of the triangle is 5 units.
Reason : (greatest side)^2 = (hypotenuse)^2 = (perpendicular)^2 + (base)^2.

Ans : (a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

Both Assertion and Reason are correct and Reason is the correct explanation of Assertion.

$$\text{Greatest side}=\sqrt{(3)^2+(4)^2}=5 \text{ units.}$$

6. **Assertion :** If $\cos A + \cos^2 A = 1$ then $\sin^2 A + \sin^4 A = 2$.
Reason : $1 - \sin^2 A = \cos^2 A$, for any value of A.

Ans : (d) Assertion (A) is false but reason (R) is true.

$$\begin{aligned} \cos A + \cos^2 A &= 1 \\ \cos A &= 1 - \cos^2 A = \sin^2 A \\ \sin^2 A + \sin^4 A &= \cos A + \cos^2 A = 1 \\ \sin^2 A + \sin^4 A &= 1 \end{aligned}$$

7. **Assertion :** In a right angled triangle, if $\cos\theta = \frac{1}{2}$ and $\sin\theta = \frac{\sqrt{3}}{2}$, then $\tan\theta = \sqrt{3}$

Reason : $\tan\theta = \frac{\sin\theta}{\cos\theta}$

Ans : (a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

Both Assertion and Reason are correct and Reason is the correct explanation of Assertion.

$$\tan\theta = \frac{\sqrt{3}}{2} \times 2 = \sqrt{3}$$

8. **Assertion :** The value of $\sin\theta\cos(90-\theta) + \cos\theta\sin(90-\theta)$ equals to 1.
Reason : $\tan\theta = \sec(90-\theta)$

Ans : (c) Assertion (A) is true but reason (R) is false.

$$\begin{aligned} \sin\theta \cdot \cos(90-\theta) + \cos\theta\sin(90-\theta) \\ &= \sin\theta \cdot \sin\theta + \cos\theta \cdot \cos\theta \\ &= \sin^2\theta + \cos^2\theta = 1 \end{aligned}$$

and, $\tan\theta = \cot(90-\theta)$