Preparing Students for
PISA*

Mathematical Literacy

Teacher's Handbook

Prince
Edward
Island
CANADA

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Based almost entirely on the Organisation for Economic Co-operation and Development document *Sample Tasks from the PISA 2000 Assessment: Reading, Mathematics and Scientific Literacy* © "OECD (2002). Reproduced by permission of the OECD."
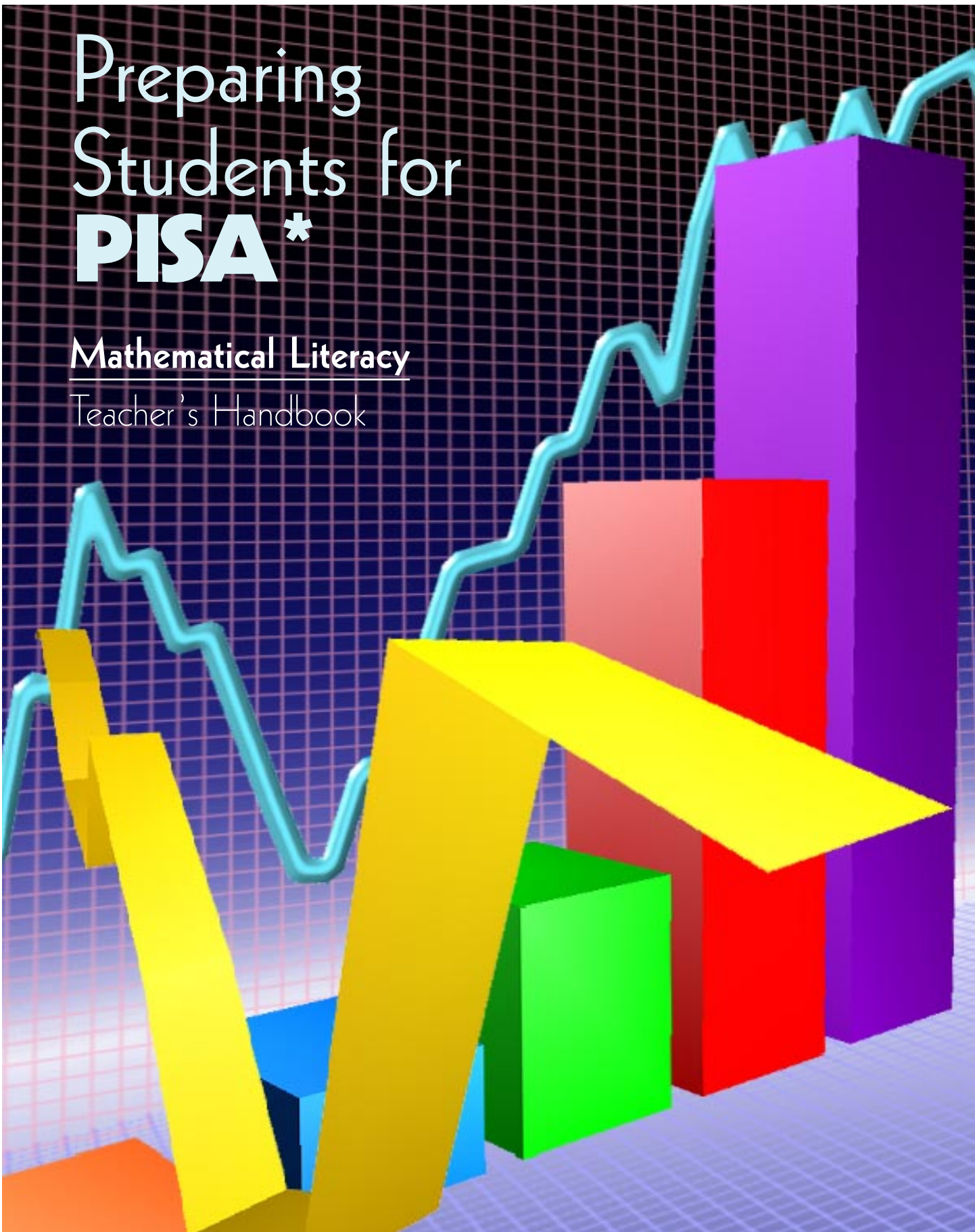
Introduction

PISA — Programme for International Student Assessment

PISA is a collaborative effort on the part of the member countries of the OECD (Organisation for Economic Co-operation and Development) to measure how well 15-year-olds are prepared to meet the challenges of today's knowledge societies. Over 40 countries, including Canada, and more than a quarter of a million students participate in this international assessment that occurs every three years. PISA assesses three domains: reading literacy, mathematical literacy, and scientific literacy.

How PISA Works

A sample of 15-year-old students is randomly chosen from selected schools in each country for the PISA assessment. PISA is a two-hour pen-and-paper assessment with both multiple-choice questions and questions requiring students to construct their own answers. Students and principals also complete a questionnaire. Each assessment examines one domain in depth, and the other two domains provide a summary profile of skills. Reading literacy was examined in depth in 2000, mathematical literacy will be examined in depth in 2003, and scientific literacy will be examined in depth in 2006.

Significance of PISA

The internationally comparable evidence on student performance can assist jurisdictions to bring about improvements in schooling to better prepare young people to enter a society of rapid change and global interdependence. As well, it can provide directions for policy development, for curricular and instructional efforts, and for student learning. Coupled with appropriate incentives, it can motivate students to learn better, teachers to teach better, and schools to be more effective. PISA represents an unprecedented effort to achieve comparability of results across countries, cultures, and languages.

Canadian Context

Approximately 30 000 15-year-old students from more than 1000 schools across Canada took part in the first administration of PISA in 2000. A large Canadian sample was drawn so that information could be provided at both national and provincial levels. Canadian students performed well in the global context, ranking second in reading, sixth in mathematics, and fifth in science. The performance of the students in the Atlantic provinces was above the international average, but well below the Canadian average.
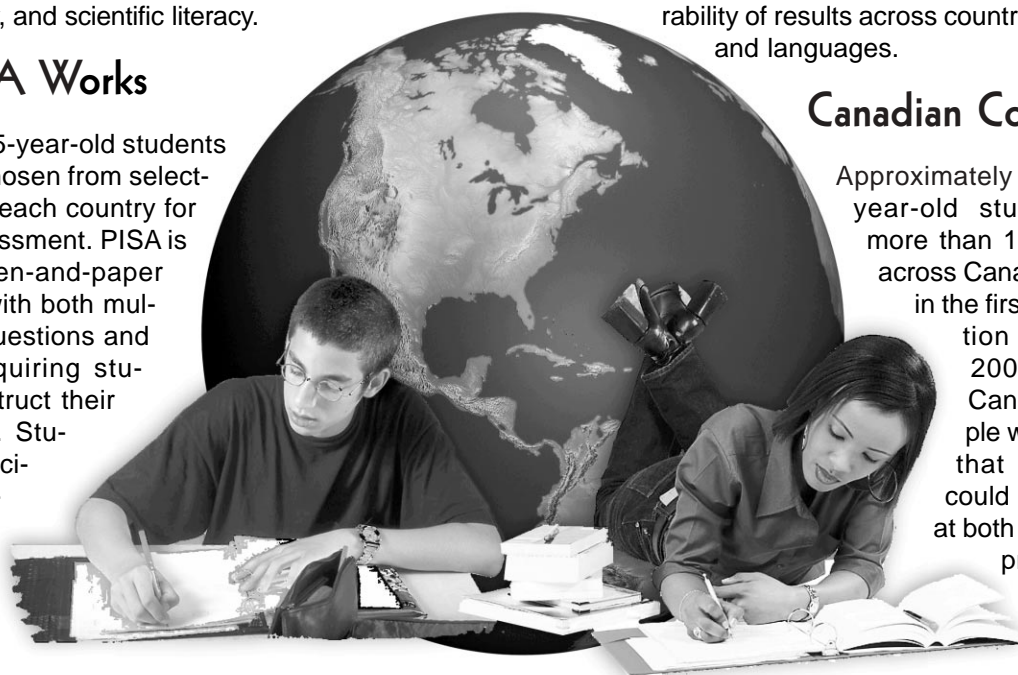
Preparing Atlantic Canadian Students for PISA

In preparation for the next PISA assessment, two documents have been prepared, one for teachers and another for students. In this document for teachers, there are two examples for whole-class discussion and two sample tasks with answers and scoring criteria. In the companion document for students, the sample tasks are also provided but without answers and scoring criteria. These two documents are published to enable students, with the help of their teachers, to attain a clear understanding of the assessment and how it is scored and to help ensure more confident and successful participation. There is also a pamphlet for parents to raise awareness of the purpose, methodology, and significance of PISA.

Mathematical Literacy

Mathematical literacy is defined in PISA as the capacity to identify, to understand, and to engage in mathematics and make well-founded judgments about the role that mathematics plays, as needed for an individual's current and future private life, occupational life, social life with peers and relatives, and life as a constructive, concerned, and reflective citizen.

Suggestions for Teachers

Teachers may decide how best to use these sample tasks. Teachers may begin by leading a detailed discussion of each task and the scoring criteria, or they may ask students to attempt each task and then discuss it afterward with the students. Additional tasks are available in two documents, *Sample Tasks from the PISA 2000 Assessment* and *Measuring Student Knowledge and Skills* found on the OECD web site www.pisa.oecd.org.

Sample Tasks

There are five sample tasks in this booklet:

Task 1, Apples: patterns and relationships
Task 2, Antarctica: measurement and similarities
Task 3, Racing Car: patterns and relations
Task 4, Triangles: space and shape
Task 5, Farms: geometry (shape) and measurement

How to Use This Document

• Engage your students in each of the five tasks in the document. This may be done as a whole class discussion or by asking students to work on the tasks individually (a companion document is provided for individual student use).

• Scoring criteria, according to PISA guidelines, are given for each question in each task. The criteria are the same as those used by PISA markers to mark the actual assessment. Examine the scoring criteria to see how each question within the task will be marked. Review the scope of acceptable answers with your students.

• Read the information that accompanies each task. This information may also be used to stimulate students to think about alternative strategies to complete each task, to help students become comfortable with the way PISA questions are formatted and classified, and to show students that the mathematics in the PISA assessment is aligned to what they are learning in their mathematics classroom.

• Use the five tasks in this document when planning for a unit of work on a specific topic in the Atlantic Canada curriculum. After reading the information that accompanies each task, reflect on your own teaching practices and what you know about your students. Try to incorporate the tasks in this document into your instructional and assessment plans.

• Remind students that if they are chosen as part of the PISA sample, partial marks are given for partially correct answers and encourage them to take the assessment seriously and strive for excellence.

PISA's "levels of mathematical competency"

• PISA organizes mathematical competency into three classes:

1) reproduction, definitions, and computations;

2) connections and integration for problem solving;

3) mathematization, mathematical thinking, generalization, and insight.

Competency Class	Description
1) Reproduction, definitions, and computations	assesses students' knowledge of • facts • representing, recognizing equivalents • recalling mathematical objects and properties • performing routine procedures • applying standard algorithms • developing technical skills
2) Connections and integration for problem solving	assesses students' abilities to • make connections between the different strands and topics in mathematics • integrate information in order to solve simple problems • make connections among the different representations • decode and interpret symbolic and formal language and understand its relation to natural language
3) Mathematization, mathematical thinking, generalization, and insight	assesses students' abilities to • recognize and extract the mathematics embedded in the situation (to mathematize) • use mathematics to solve the problem • analyse, interpret, and develop their own models and strategies • make mathematical arguments, including proofs and generalizations.

Assessment-taking Strategies

Some strategies for students to consider:

- Always read the information for each task carefully.

- Reread each task question and any accompanying text before attempting an answer.

- Give each question a try, even when you're not sure. Remember partial value is given for partially correct answers.

- Interpretive, reflective, and evaluative questions are questions that begin with Why ... ? Why do you think ... ? How do you know ... ? One- or two-word answers are insufficient. Reasons, usually with reference to the task, are required; often the word "because" is used in the response.

- Non-continuous texts such as graphs (Task 3, Racing Car), charts, and tables (Task 1, Apples) provide a valid reading situation. Study the axes and determine the purpose of the text before answering the questions. For example, in Racing Car the purpose of the text is to show changing speeds as a car races along a curved race track; the horizontal axis shows the distance (km) along the track from the starting line; the vertical axis, the speed (km/h).

- Develop a methodical process of elimination of the alternatives in multiple choice questions. When the list is narrowed to the best possibilities, choose one; there is no extra penalty for wrong choices.

Tasks

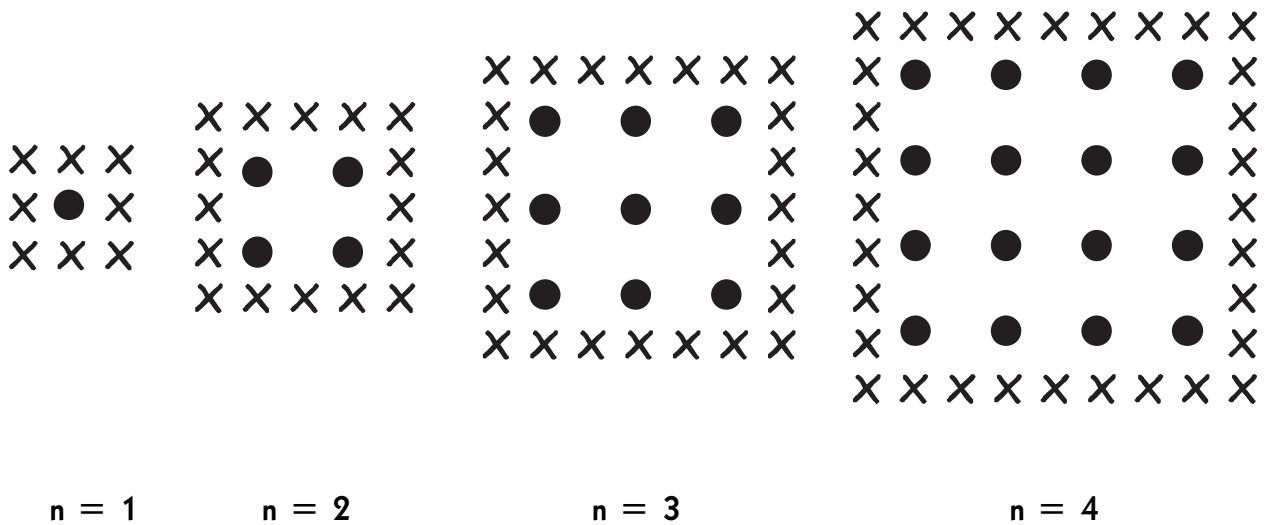## Mathematics Task 1
APPLES

A farmer plants apple trees in a square pattern. In order to protect the trees from the wind he plants evergreens all around the orchard.

The diagram below illustrates the pattern of apple trees and evergreens for any number (*n*) of rows of apple trees:

✗ = evergreen
● = apple tree

n = 1   n = 2   n = 3   n = 4

Question 1

Complete the following table:

n	Number of Apple Trees	Number of Evergreens
1	1	8
2	4	
3		
4		

7		

Total value: 2 points

■ **Give 1** point for all five correct entries in the first four rows of the table:
apple trees 9, 16; evergreens 16, 24, 32.
■ **Give 1** point for both correct entries in row 7: **49; 56**.

The student can get either 2, 1, or 0 points.

Question 2

There are two formulae you can use to calculate the number of apple trees and the number of evergreens for the pattern described above:

Let n be the number of rows of apple trees.
Then, the number of apple trees = _____
and the number of evergreens = _____

There is a value for n for which the number of apple trees equals the number of evergreens.
Find the value of n and show your method of calculating this.

Total value: 3 points

■ **Give 1** point for "n^2" expressed in symbols *or* in words such as
"**the square of the number of rows of apple trees**."
■ **Give 1** point for "$8n$" expressed in symbols *or* in words such as
"**eight times the number of rows of apple trees**."
■ **Give 1** point for the correct answer "$n = 8$" *or* any other form that
indicates the answer (sentence, circled '8', etc. ...).
■ **Give 0.5** points for the correct answer "**8**" for the third part of the question,
but also the inadmissable answer "**0**." So, if the student thinks that $n = 0$ *and* 8, the student
receives 0.5 points.

The student can get either 3, 2.5, 2, 1.5, 1, 0.5, or 0 points.

Question 3

Suppose the farmer wants to make a much larger orchard with many rows of trees.
As the farmer makes the orchard bigger, which will increase more quickly: the number
of apple trees or the number of evergreens? Explain how you found your answer.

Total value: 2 points

■ **Give 2 points** for the correct answer "**the number of apple trees**" as long as it includes
a plausible explanation that includes algebraic reasoning. For example:

- Apple trees = n x n and evergreens = 8 x n. Both formulae have a factor n, but apple trees
  have another n, which will get larger while factor 8 stays the same. The number of apple
  trees increases more quickly.
- The number of apple trees increases faster because that number is squared instead of
  being multiplied by 8.

1) The number of apple trees is quadratic. The number of evergreens is linear. So apple trees
   will increase faster.
2) The answer uses a graph to demonstrate that n^2 exceeds $8n$ after $n = 8$.

■ **Give 1.5 points** for the correct answer "**the number of apple trees**," but the reasoning is
based on specific examples or on extending the table. For example:

- The number of trees will increase more quickly because, if we use the table (previous page),
  we find that the number of apple trees increases faster than the number of evergreen trees.
  This happens especially after the number of apple trees and the number of evergreens are
  the same.

■ **Give 1.5 points** for the correct answer "**the number of apple trees**" and show
some evidence that the relationship between n^2 and $8n$ is understood, but not clearly
expressed. For example:

- Apple trees after $n > 8$.
- After 8 rows, the number of apple trees will increase more quickly than evergreens.
- Evergreens until you get to 8 rows, then there will be more apple trees.

■ **Give 1 point** for the correct answer, but insufficient or wrong explanation or no explanation.
For example:

- Apple trees.
- Apple trees because they are populating the inside, which is bigger than just the perimeter.
- Apple trees because they are surrounded by evergreens.

■ **Give 0 points** for the incorrect answer.

The student can get either 2, 1.5, 1, or 0 points.

Information About Task 1 - Apples

Competency Class for Questions in this Task

According to PISA, questions 1 and 2 are competency class 2 (connections and integration for problem solving) questions. Question 3 in this task is a competency class 3 (mathematization, mathematical thinking, generalization, and insight) question. (See p. 5 for descriptors for PISA's classes of mathematical competency.)

Connection to the Atlantic Canada Curriculum

This task deals with patterns and relationships, which align with the Atlantic Canada Curriculum, especially General Curriculum Outcome (GCO) C. The task supports Unit 3.5 for teachers who use the resource *Mathematical Modeling*, *Book 1*, or *Constructing Mathematics*, *Book 1* and could be used as an additional problem when students are working with page 132 in *Mathematical Modeling*, *Book 1*, or page 135 in *Constructing Mathematics*, *Book 1*.

The following is a list of the Specific Curriculum Outcomes (SCOs) that each of the questions in the task addresses:

Question 1

For grade 9 students

9C1 represent patterns and relationships in a variety of formats and use these representations to predict and justify unknown values

9C3 construct and analyse tables and graphs to describe how changes in one quantity affect a related quantity

9C5 explain the connections among different representations of patterns and relationships

For grade 10 students

10A2 analyse graphs or charts of situations to derive specific information

10C8 identify, generalize, and apply patterns

10C9 construct and analyse graphs and tables relating two variables

10C10 describe real-world relationships depicted by graphs, tables of values, and written descriptions

Question 2

For grade 9 students

9C1 represent patterns and relationships in a variety of formats and use these representations to predict and justify unknown values

9C3 construct and analyse tables and graphs to describe how changes in one quantity affect a related quantity

9C8 solve and create tasks involving linear equations and inequalities

Note: There is no outcome in grade 9 for solving a second-degree equation. However, students reason the solution to be 8 by other methods, thus, perhaps, addressing outcome B15.

9B15   select and use appropriate strategies in problem situations

For grade 10 students

10B1   model and express the relationships between arithmetic operations and operations on algebraic expressions and equations

10C2   model real-world phenomena with linear, quadratic, exponential, and power equations and linear inequalities

10C8   identify, generalize, and apply patterns

10C9   construct and analyse graphs and tables relating two variables

10C10 describe real-world relationships depicted by graphs, tables of values, and written descriptions

10C17 solve problems using graphing technology

10C26 solve quadratic equations by factoring

10C28 explore and describe the dynamics of change depicted in tables and graphs

Question 3

For grade 9 students

9B1   model, solve, and create problems involving real numbers

9B6   determine the reasonableness of results in problem situations involving square roots, rational numbers, and numbers written in scientific notation

9B15   select and use appropriate strategies in problem situations

9C1   represent patterns and relationships in a variety of formats and use these representations to predict and justify unknown values

9C3   construct and analyse tables and graphs to describe how changes in one quantity affect a related quantity

9C5   explain the connections among different representations of patterns and relationships

For grade 10 students

10A6   apply properties of numbers when operating upon expressions and equations

10B1   model and express the relationships between arithmetic operations and operations on algebraic expressions and equations

10C2   model real-world phenomena with linear, quadratic, exponential, and power equations and linear inequalities

10C15 develop and apply strategies for solving problems

10C16 interpret solutions to equations based on context

10C20 evaluate and interpret non-linear equations using graphing technology

Additional Information for Each Question

There are three questions in this task. The questions originated from the sample task published by PISA. Small changes have been made so that the question aligns more closely with classroom practice of mathematics teachers in Atlantic Canada.

Question 1

Students are required to complete a table of values. Some of the values are given to help students get started. Students could complete the table for the first four rows using counting techniques or by using patterns. From looking only at the numbers in the table it may not be clear to teachers which method students used to obtain their numbers. The fifth row in the table, where $n = 7$ is a jump in the list of n-values, giving the student an opportunity to use a pattern, rather than just count, to obtain an answer. Students need to work with given models and to relate two different representations (pictorial and tabular) of two relationships (one quadratic and one linear) in order to extend the pattern. In the classroom, if this task is used as an instructional tool, large-group discussion would indicate the different strategies, or thinking, students used to complete the table. For example, some may simply continue to make diagrams and count, while others may follow a recursive pattern and fill in the table based on the previous number placed above. Still others, when they get to the fifth row may use a relational pattern and complete the row in the table by squaring the n-value in the first case and multiplying the n-value by 8 in the second case.

Question 2

The second question gives students the opportunity to state the relational pattern in words or mathematical symbols. The question requires students to find the value for n when the number of apple trees is the same as the number of evergreen trees. Full value is given for any strategy to solve the question that leads to a correct answer, as long as the student's thinking and use of the strategy are made clear (see the marking guide).

The diagram and the table suggest that there are always more evergreens than apple trees.

Question 3

The third question requires students to show insight into mathematical functions by comparing the growth of a linear function with that of a quadratic function. The question asks students to extend the table or the diagram or to evaluate the relational-pattern expressions (algebraically) to determine which set of numbers (apples or evergreens) grows more quickly and to explain their answer.

Mathematics Task 2
ANTARCTICA

Question 1

Estimate the area of Antarctica using the map scale. Show your work and explain how you made your estimate. (You can draw over the map if it helps you with your estimation.)

Total value: 2 points

■ **Give 2** **points** for answers that are estimated by drawing a square or rectangle—between 12 000 000 km^2 and 18 000 000 km^2.

■ **Give 2** **points** for answers that are estimated by drawing a circle—between 12 000 000 km2 and 18 000 000 km^2.

■ **Give 2** **points** for answers that are estimated by adding (and/or subtracting) areas of several regular geometric figures—between 12 000 000 km^2 and 18 000 000 km^2.

■ **Give 2** **points** for answers that are estimated by other correct methods—between 12 000 000 km^2 and 18 000 000 km^2.

■ **Give 1.5** **points** for answers that are correct—between 12 000 000 km^2 and 18 000 000 km^2—work shows effort that might lead to a correct answer but no calculations are shown. The following scores are for answers that use a correct method but give an incorrect or incomplete result.

■ **Give 1** **point** for answers that are estimated by drawing a square or rectangle—correct method—but incorrect or incomplete answer. For example:

• Draws a rectangle and multiplies width by length, but the answer is an overestimate or an underestimate (e.g., 18 200 000).
• Draws a rectangle and multiplies width by length, but the number of zeros is incorrect (e.g., 4000 x 3500 = 140 000).
• Draws a rectangle and multiplies width by length, but forgets to use the scale to convert to square kilometres (e.g., 12 cm x 15 cm = 180).
• Draws a rectangle and states the area is 4000 km x 3500 km. No further working out.

■ **Give 1 point** for answers that are estimated by drawing a circle—correct method—but incorrect or incomplete result.

■ **Give 1 point** for answers that are estimated by adding areas of several regular geometric figures—correct method—but incorrect or incomplete result.

■ **Give 1 point** for answers that are estimated by other correct methods, but incorrect or incomplete result.

■ **Give 0 points** for answers that show the perimeter instead of area. For example:
• 16 000 km as the scale of 1000 km would go around the map 16 times

■ **Give 0 points** for incorrect answers. For example:
• 16 000 km (no working out is shown, and the result is incorrect)

The student can get either 2, 1.5, 1, or 0 points.

Information about Task 2 - Antarctica

Competency Class for Questions in this Task

This question is considered "competency class 2" because of the "connections and integration for problem solving." (See p. 5 for descriptors of PISA's classes of mathematical competency.)

Connection to the Atlantic Canada Curriculum

This task deals with measurement and similarity and thus, in part, addresses GCO D. For teachers using the resource *Mathematical Modeling, Book 1*, or *Constructing Mathematics, Book 1*, this question could be given to students early in Unit 5 while looking at similarity or in Unit 6 when looking at surface area.

The following is a list of the Specific Curriculum Outcomes (SCO) that the question in the task addresses.

Question 1:

For grade 9 students

9C1   represent patterns and relationships in a variety of formats and use these representations to predict and justify unknown values

9B15   select and use appropriate strategies in problem situations

9D4   estimate, measure, and calculate dimensions, volumes, and surface areas of pyramids, cones, and spheres in problem situations

For grade 10 students

10C10 describe real-world relationships depicted by graphs, tables of values, and written descriptions

10C15 develop and apply strategies for solving problems

10D1 determine and apply formulas for perimeter, area, surface area, and volume

10D2 apply the properties of similar triangles

10D8 solve problems involving similar triangles and right triangles

10D13 demonstrate an understanding of the concepts of surface area and volume

Additional Information for Each Question

This task originated from the sample questions published by PISA.
No changes have been made. The task deals with how a map scale can be used to estimate the surface area of a whole continent.

Question 1

The question asks students to estimate the area of the Antarctic continent using a map scale. There are several ways that students could go about answering this question. For example, students could draw a rectangle around the map of the continent then use the scale graph and similarity properties to determine the dimensions of the rectangle. Next, they could estimate the area of smaller rectangles drawn over non-land mass areas and subtract to find an estimate of the part that is land. Students could also place a clear sheet of graph paper over the drawing, or trace the continent onto a sheet of graph paper, and using the squares on the graph paper, the measurements on the scale graph, and approximate calculations, determine the estimate.

Mathematics Task 3
RACING CAR

Question 1

What is the approximate distance from the starting line to the beginning of the longest straight section of the track?
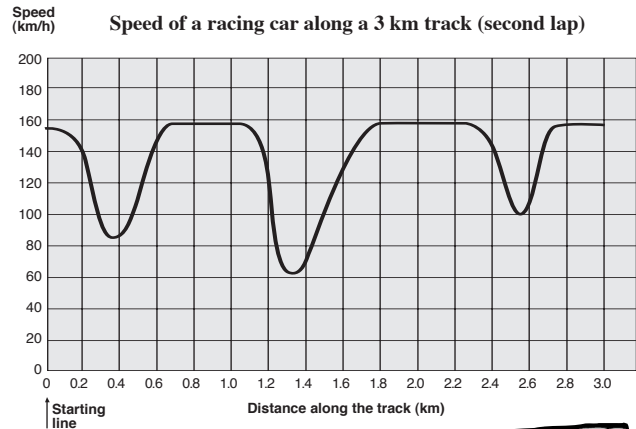
A. 0.5 km   B. 1.5 km   C. 2.3 km   D. 2.6 km

Total value: 1 point

■ **Give 1 point** for the correct answer

B 1.5 km.

■ **Give 0 points** for other answers.

The student can get either 1 or 0 points.

Speed (km/h)   **Speed of a racing car along a 3 km track (second lap)**

Distance along the track (km)

Starting line

This graph shows how the speed of a racing car varies along a flat 3 kilometre track during its second lap.
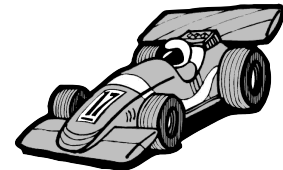
Question 2

Where is the lowest speed recorded during the second lap?

A. at the starting line
B. at about 0.8 km
C. at about 1.3 km
D. half way around the track

Total value: 1 point

■ **Give 1 point** for the correct answer **C 1.3 km**.
■ **Give 0 points** for other answers.

The student can get either 1 or 0 points.

Question 3

What can you say about the speed of the car between the 2.6-km and 2.8-km marks?

A. The speed of the car remains constant.
B. The speed of the car is increasing.
C. The speed of the car is decreasing.
D. The speed of the car cannot be determined from the graph.

Total value: 1 point

■ **Give 1** point for the correct answer **B The speed of the car is increasing**.
■ **Give 0** points for other answers.

The student can get either 1 or 0 points.

Question 4

Along which one of the tracks below was the car driven to produce the speed graph shown on the previous page?

O = Starting point
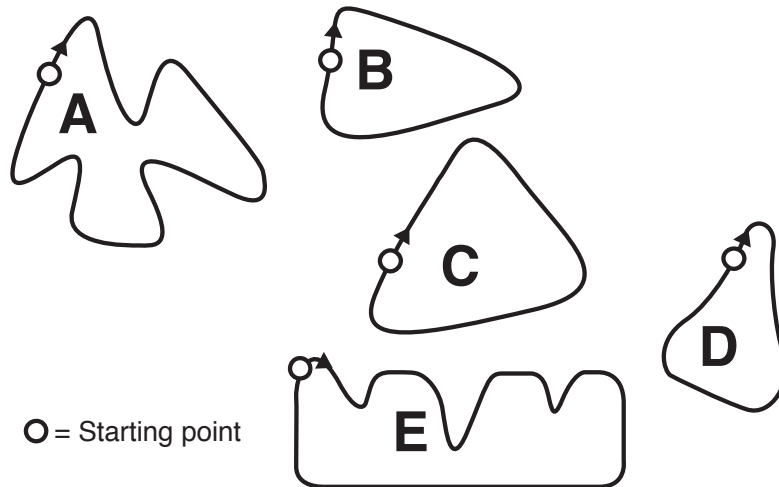
Total value: 1 point

■ **Give 1 point** for the correct answer: **Track B**.
■ **Give 0 points** for other answers.

The student can get either 1 or 0 points.

Information about Task 3, Racing Car

Competency Class for Questions in This Task

According to the PISA descriptors, Questions 1 and 4 are considered "competency class 2" because of the "connections and integration for problem solving." Questions 2 and 3 are "competency class 1" because they involve either reproduction, definitions, or computations. (See p. 5 for descriptors of PISA's classes of mathematical competency.)

Connection to the Atlantic Canada Curriculum

This task deals with patterns and relations and thus mostly addresses GCO C. For teachers using the resource *Mathematical Modeling, Book 1*, or *Constructing Mathematics, Book 1*, this question could be given to students while working in Unit 4.1, p. 160 in *Mathematical Modeling*, and p. 160 in *Constructing Mathematics*. Actually there is a question almost identical to this in the *Mathematical Modeling* text, so it may be better to give this task to students after that unit as a review or on a test or assignment.

The following is a list of the SCOs that each of the questions in the task addresses.

Questions 1–4 inclusive

For grade 9 students

9C2 interpret graphs that represent linear and non-linear data

9C3 construct and analyse tables and graphs to describe how changes in one quantity affect a related quantity

9C5 explain the connections among different representations of patterns and relationships

For grade 10 students

10A2 analyse graphs or charts of situations to derive specific information

10C8 identify, generalize, and apply patterns

10C10 describe real–world relationships depicted by graphs, tables of values, and written descriptions

10C28 explore and describe the dynamics of change depicted in tables and graphs

Additional Information for Each Question

This task originated from the sample tasks published by PISA. No changes have been made. The task deals with how students interpret graphs of real-world phenomena. Often students think that graphs are just pictures of situations and don't think of them as pictures of how the variables in the situation are related. This task includes four questions.

Question 1

The first question (a selected response question) asks students to interpret the speed-versus-distance graph. They must look for where they think the longest straightaway would be on the track. To find this from the graph, they would have to locate a section that shows the car accelerating to maximum speed then continuing at that speed for a longer period of time. They have to determine the distance from the beginning of that straightaway to the starting line. This question then asks students to select the most appropriate answer from four plausible answers.

Question 2

The second question (also a selected response) asks students to read the speed that is the lowest on the graph.

Question 3

The third question (also a selected response) asks students to examine the graph in a particular time interval then select the most appropriate answer with respect to the speed of the car during that interval.
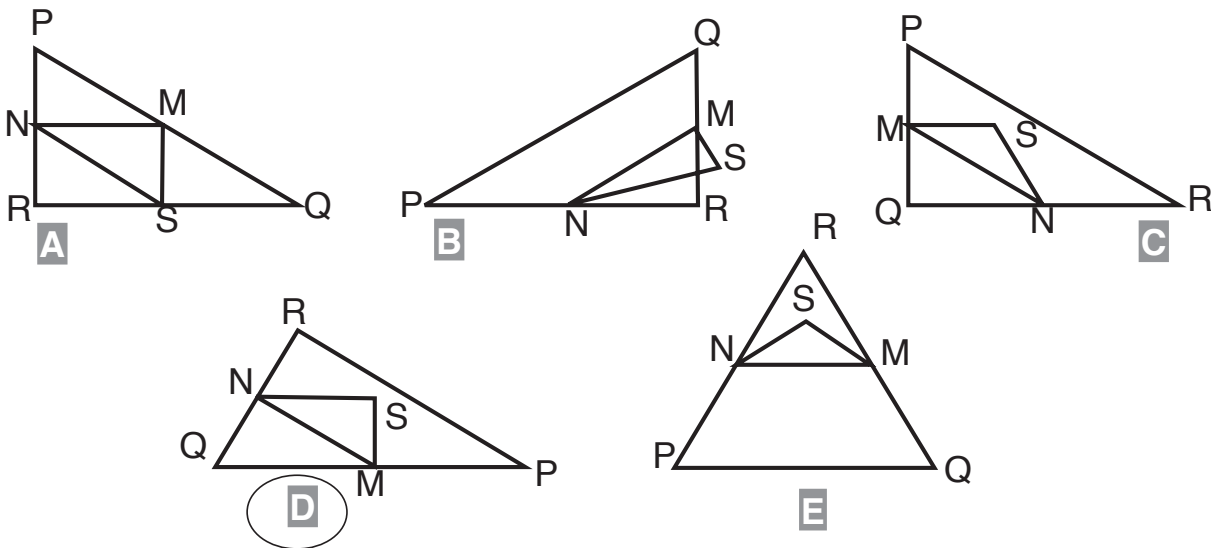
Question 4

The fourth question asks students to examine five plausible designs for a race track and select the one that best matches the graph given at the beginning of the task. In order to complete this successfully, students must interpret the slowing down for curves and accelerating through the curves to straightaways and match that to the graph.

Question 1

Circle the one figure below that fits the following description:

Triangle PQR is a right triangle with the right angle at R. The segment RQ is less than the segment PR. M is the midpoint of the segment PQ, and N is the midpoint of the segment QR. S is a point inside the triangle. The segment MN is greater than the segment MS.

Total value: 1 point
■ **Give 1** point for the correct answer: D.
■ **Give 0** points for other answers.
The student can get either 1 or 0 points.

Question 2

Given that P, Q, and R are the midpoints of the three sides of the ΔABC respectively.
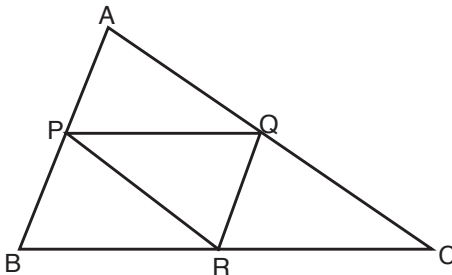
Determine at least one pair of triangles that must be congruent, and justify your decision.

Total value: 2 points

■ **Give 1** point for **naming a correct pair of congruent triangles** from the four congruent triangles: ΔAPQ ≅ ΔPBR ≅ ΔQRC ≅ ΔPRQ and indicating (in symbols or words) that they are congruent.

■ **Give 0.5** **points** for naming a correct pair but using only an equal sign (or the word equal) instead of a congruent sign (or the word congruent).

■ **Give 1** **point** for correctly justifying why the triangles are congruent using transformations or stating the minimum sufficient conditions required and why each is justified. For example:

• For the pair of triangles $\triangle APQ$ and $\triangle PBR$, since P is a midpoint, then it can be proved that AP = BP = RQ, and that RQ = ½AB, so that, if $\triangle APQ$ is translated such that P→B, then Q→R, and A→P, then it would map onto $\triangle PBR$, and the two triangles would be congruent. Similar arguments can be made for $\triangle APQ$ and $\triangle QRC$, and for $\triangle QRC$ and $\triangle PBR$. Rotation properties would have to be used to get $\triangle PRQ$ congruent to any of the other named triangles.

• Students could say that each of the midline segments could be proved parallel to the corresponding side of the $\triangle ABC$, thus causing several pairs of congruent corresponding angles and alternate interior angles. So, for example, the corresponding angles ∠APQ and ∠PBR are congruent, as well the corresponding angles ∠PAQ and ∠BPR are congruent. Since the sides AP and BP have the same length, then the triangles $\triangle APQ$ and $\triangle PBR$ are congruent because of the property called ASA. Students could also use the SAS or the SSS properties to establish congruence.

■ **Give 0.5** points to students who make a good attempt to justify why two correct triangles are congruent.

The student can get either 2, 1.5, 1, 0.5, or 0 points.

Information about Task 4, Triangles

Competency Class for Questions in This Task

Question 1 is considered "competency class 1" because of the "reproduction, definitions, or computations" that are required. The second question was not in the original task, but would be classified as "competency class 3" because of the "mathematization, mathematical thinking, and the insight" that would be required. (See p.5 for descriptors of PISA's classes of mathematical competency.)

Connection to the Atlantic Canada Curriculum

This task deals with space and shape and thus mostly addresses GCO E, more specifically at the grade 9 level. For teachers using the resource *Mathematical Modeling, Book 1*, or *Constructing Mathematics, Book 1*, this question could be given to students perhaps as an introduction to Unit 5.2 where teachers might use it as a review of grade 9 geometry and as an introduction to the need for proof. Teachers may want to give this task during the last section in Unit 6 in the *Mathematical Modeling* book, where students are working with triangles and parts of triangles.

The following is a list of the Specific Curriculum Outcomes (SCO) that each of the questions in the task addresses.

Question 1

For grade 9 students
9E1   investigate and demonstrate an understanding of the minimum sufficient conditions to produce unique triangles

9E2   investigate and demonstrate an understanding of the properties of, and the minimum sufficient conditions to guarantee congruent triangles

9E3   make informal deductions using congruent triangle and angle properties

For grade 10 students
10E1   explore properties of, and make and test conjectures about two- and three-dimensional figures

10E8   use inductive and deductive reasoning when observing patterns, developing properties, and making conjectures

10E9   use deductive reasoning and construct logical arguments and be able to determine, when given a logical argument, if it is valid

Additional Information for Each Question

This task originated with the sample tasks published by PISA. The task deals partly with perception and partly with geometrical reasoning and justification. There are two questions.

Question 1

The first question asks students to read a description of a two-dimensional shape and match the description to a picture that represents the shape. Students have to demonstrate an understanding of geometric ideas such as position, angle size, length, midpoint, and the naming of shapes. The question type is selected response.

Question 2

The second question, which was not part of the original PISA task, asks students to use their understanding of what is required geometrically to determine that two triangles are congruent and to state and justify their argument that the triangles are congruent. The student may pick any one pair of triangles from several that are congruent.

Mathematics Task 5
FARMS
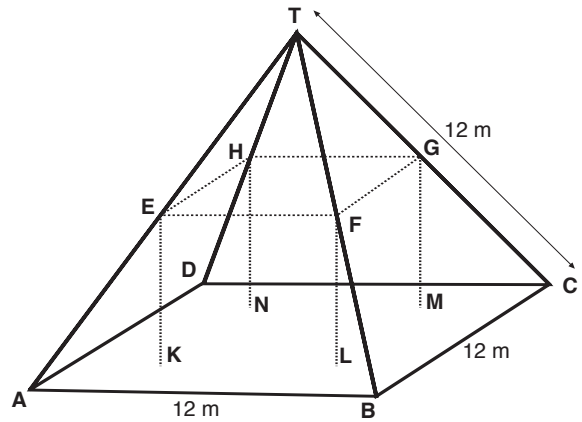
Below you see a student's mathematical model of a farmhouse roof with measurements.
The attic floor, ABCD in the model, is a square. The beams that support the roof are the edges of a rectangular prism, EFGHKLMN. E is the middle of AT, F is the middle of BT, G is the middle of CT, and H is the middle of DT. All the edges of the pyramid in the model have length of 12 m.

Question 1

Calculate the area of the attic floor ABCD.

The area of the attic floor ABCD = _____
Total value: 1 point

■ **Give 1** point for the correct answer: **144 m^2**.

■ **Give 0.5** point for the 144, expressed with no units.

The student can get either 1, or 0.5, or 0 points.

Question 2

Calculate the length of EF, one of the horizontal edges of the block.

The length of EF = ____
Total value: 1 point

■ **Give 1** point for the correct answer: **6 m**.

■ **Give 0.5** point for the 6, expressed with no units.

Question 3

If the measurement of the base of the pyramid in the model did not change, but the edges AT, BT, CT, and DT were all 15 m and E, F, G, and H remained as midpoints, would the length of EF change? Justify your answer.

Total value: 2 points

■ **Give 1** point for the correct answer: **"the length EF would not change**."

■ **Give 1** point for the correct justification. The justification should include reference to the fact that EF would have to be one-half of the base-length AB, because of the properties of similar triangles (ratios of corresponding sides of similar triangles are proportional). The student's solution does not have to state the property, but show use of it. For example, many students might say "since the triangles are similar and E and F are midpoints then EF must be one-half of AB." The word "similar" must be used correctly, the concept of "midpoint" must be applied properly, and the ratio of corresponding sides is "one-half" must be clear.

■ **Give 0.5** points for a justification that is almost complete but missing one key element. You are still convinced that the student thinks that length EF is still the same for a solid mathematical reason, not just a guess.

The student can get either 2, 1.5, 1, or 0 points.

Information About Task 5-Farms

Competency Class for Questions in this Task

Question 1 is considered "competency class 1" because of the "very procedural computation." Question 2 is considered "competency class 2" because of the "connections and integration for problem solving" involved in the solution to the question. The student is expected to link information in verbal and symbolic form to a diagram. Question 3, however, would be considered "competency class 3" because the student would have to analyse, perhaps reflect, then make mathematical arguments to justify a decision. (See p.5 for descriptors of PISA's classes of mathematical competency.)

Connection to the Atlantic Canada Curriculum

This task about a farm building deals with geometry (shape) and measurement. The measurement aspect deals with area and similarity properties, thus the task mostly addresses GCO D. For teachers using the resource *Mathematical Modeling*, *Book 1*, or *Constructing Mathematics*, *Book 1*, this question could be given to students during Unit 5 or 6 when the instructional activities are investigating or applying similarity properties and/or area applications.

The following is a list of the Specific Curriculum Outcomes that each of the questions in the task addresses:

Question 1

For grade 9 students

9D4   estimate, measure, and calculate dimensions, volumes, and surface area of pyramids, cones, and spheres in problem situations

For grade 10 students

10D1   determine and apply formulas for perimeter, area, surface area, and volume

Question 2

For grade 9 students

9D4   estimate, measure, and calculate dimensions, volumes, and surface area of pyramids, cones, and spheres in problem situations
9D5   demonstrate an understanding of and apply proportions within similar triangles

For grade 10 students

10D1   determine and apply formulas for perimeter, area, surface area, and volume
10D2   apply properties of similar triangles
10D8   solve problems involving similar triangles and right triangles
10E1   explore properties of and make and test conjectures about two- and three- dimensional figures
10E2   solve problems involving polygons and polyhedra

Question 3

For grade 9 students

9D4   estimate, measure, and calculate dimensions, volumes, and surface area of pyramids, cones, and spheres in problem situations
9D5   demonstrate an understanding of and apply proportions within similar triangles

For grade 10 students

10D1   determine and apply formulas for perimeter, area, surface area, and volume
10D2   apply properties of similar triangles
10D8   solve problems involving similar triangles and right triangles
10E1   explore properties of and make and test conjectures about two- and three-dimensional figures
10E2   solve problems involving polygons and polyhedra
10E9   use deductive reasoning and construct logical arguments and be able to determine, when given a logical argument, if it is valid

Additional Information for Each Question

This task originated with the sample questions published by PISA. Small changes have been made so that the student would not only have to report the numerical answer, but also have to give the proper unit(s). Also, a third question has been added to confirm the use of similarity properties since the answer to question 2 could easily be guessed by a student. The task now better matches the classroom practice of mathematics teachers in Atlantic Canada. The task has three questions.

Question 1

The first question asks students to calculate the area of the base of a pyramid. Some lengths for the pyramid are given to help the student determine how to calculate the area. As well, students are told that the attic floor (which is the base of the pyramid) is a square. Students could calculate the answer by simply multiplying the length by the width or by squaring the length.

Question 2

The second question gives students the opportunity to use similarity properties to calculate one of the lengths in the diagram. This task is designed to require students to work with a familiar geometric model and to link information in verbal and symbolic form to a diagram. Students need to "dis-embed" a triangle from a two-dimensional representation of a three-dimensional object, then select the appropriate information about side length relationships within similar triangles. However, since the students are required to fill in a blank with a number and a unit, it is possible for them to guess the correct answer as "one-half of 12" (thinking one-half because of the word "midpoint").

Question 3

The third question will confirm knowledge of similarity properties since students will be expected to consider different measurements than in question 2, and they will be required to justify their decision.