STUDENT SUPPORT MATERIAL

Session : 2019-20

तत् त्वं पूषन् अपावृणु
केन्द्रीय विद्यालय संगठन

CLASS X

MATHEMATICS

केन्द्रीय विद्यालय संगठन, नई दिल्ली

KENDRIYA VIDYALAYA SANGATHAN, NEW DELHI

STUDENT SUPPORT MATERIAL

Class –X

Mathematics

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KENDRIYA VIDYALAYA SANGATHAN

NEW DELHI

संतोष कुमार मल्ल, भा.प्र.से.

आयुक्त

Santosh Kumar Mall, I.A.S.

Commissioner

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A WORD TO MY DEAR STUDENTS

It gives me great pleasure in presenting the Students 'Support Material to all KV students of class X. The material has been prepared keeping in mind your needs when you are preparing for final exams and wish to revise and practice questions or when you want to test your ability to complete the question paper in the time allotted or when you come across a question while studying that needs an immediate answer but going through the textbook will take time or when you want to revise the complete concept or idea in just a minute or try your hand at a question from a previous CBSE Board exam paper or the Competitive exam to check your understanding of the chapter or unit you have just finished. This material will support you in any way you want to use it.

A team of dedicated and experienced teachers with expertise in their subjects has prepared this material after a lot of exercise. Care has been taken to include only those items that are relevant and are in addition to or in support of the textbook. This material should not be taken as a substitute to the NCERT textbook but it is designed to supplement it.

The Students 'Support Material has all the important aspects required by you; a design of the question paper, syllabus, all the units/chapters or concepts in points, mind maps and information in tables for easy reference, sample test items from every chapter and question papers for practice along with previous years Board exam question papers.

I am sure that the Support Material will be used by both students and teachers and I am confident that the material will help you perform well in your exams.

Happy learning!

Santosh Kumar Mall

Commissioner, KVS

FOREWORD

The Students' Support Material is a product of an in-house academic exercise undertaken by our subject teachers under the supervision of subject expert at different levels to provide the students a comprehensive, yet concise, learning support tool for consolidation of your studies. It consists of lessons in capsule form, mind maps, concepts with flowcharts, pictorial representation of chapters wherever possible, crossword puzzles, question bank of short and long answer type questions with previous years' CBSE question papers.

The material has been developed keeping in mind latest CBSE curriculum and question paper design. This material provides the students a valuable window on precise information and it covers all essential components that are required for effective revision of the subject.
In order to ensure uniformity in terms of content, design, standard and presentation of the material, it has been fine-tuned at KVS HQRS level.

I hope this material will prove to be a good tool for quick revision and will serve the purpose of enhancing students 'confidence level to help them perform better. Planned study blended with hard work, good time management and sincerity will help the students reach the pinnacle of success.

Best of Luck.

U.N. Khaware
Additional Commissioner (Acad.)

Mathematics

STUDENT SUPPORT MATERIAL

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- Shri Kamal Kumar, TGT(Maths),KV Sitapur.
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COURSE STRUCTURE CLASS -X

As per the CBSE Circular F.1002/CBSE/Dir. (Acad.)/Mathematics dated 10/01/2019 the two levels of Examination will be held in the subject of Mathematics in the Board Examination for class X in the year 2020

Units	Unit Name	Marks
I	NUMBER SYSTEMS	06
II	ALGEBRA	20
III	COORDINATE GEOMETRY	06
IV	GEOMETRY	15
V	TRIGONOMETRY	12
VI	MENSURATION	10
VII	STATISTICS & PROBABILTY	11
	Total	**80**

Time: 3 Hours   **Max. Marks: 80**

S. No.	Typology of Questions	Very Short Answer-Objective type (VSA) (1 Mark)	Short Answer-I (SA)(2 Marks)	Short Answer-II (SA) (3 Marks)	Long Answer (LA)(4 Marks)	Total Marks	% Weightage (approx.)
1	**Remembering:** Exhibit memory of previously learned material by recalling facts, terms, basic concepts, and answers.	6	2	2	1	20	25
2	**Understanding:** Demonstrate understanding of facts and ideas by organizing, comparing, translating, interpreting, giving descriptions, and stating main ideas	6	1	1	3	23	29
3	**Applying:** Solve problems to New situations by applying acquired knowledge, facts, techniques and rules in a different way.	5	2	2	1	19	24
4	**Analyzing :** Examine and break information into parts by identifying motives or causes. Make inferences and find evidence to support generalizations **Evaluating:** Present and defend opinions by making judgments about information, validity of ideas, or quality of work based on a set of criteria. **Creating:** Compile information together in a different way by combining elements in a new pattern or proposing alternative solutions	3	1	3	1	18	22
	Total	20x1 =20	6x2 =12	8x3=24	6x4=24	80	100

INTERNAL ASSESSMENT	20 MARKS
Pen Paper Test and Multiple Assessment(5+5)	10 Marks
Portfolio	05 Marks
Lab Practical (Lab activities to be done from the prescribed books)	05 Marks

MATHEMATICS-Basic Code (241)
QUESTION PAPER DESIGN
CLASS – X (2019-20)

Time: 3 Hours   **Max. Marks: 80**

S. No.	Typology of Questions	Very Short Answer-Objective type (VSA) (1 Mark)	Short Answer-I (SA) (2Marks)	Short Answer-II (SA) (3Marks)	Long Answer (LA) (4Marks)	Total Marks	% Weightage (approx.)
1	**Remembering:** Exhibit memory of previously learned material by recalling facts, terms, basic concepts, and answers.	5	2	5	2	32	40
2	**Understanding:** Demonstrate understanding of facts and ideas by organizing, comparing, translating, interpreting, giving descriptions, and stating main ideas	7	1	1	4	28	35
3	**Applying:** Solve problems to new situations by applying acquired knowledge, facts, Techniques and rules in a different way.	5	2	1	-	12	15
4	**Analyzing :** Examine and break information into parts by identifying motives or causes. Make inferences and find evidence to support generalizations **Evaluating:** Present and defend opinions by making judgments about information, validity of ideas, or quality of work based on a set of criteria. **Creating:** Compile information together in a different way by combining elements in a new pattern or proposing alternative solutions	3	1	1	-	8	10
	Total	20x1 =20	6x2 =12	8x3=24	6x4=24	80	100

INTERNAL ASSESSMENT	20 MARKS
Pen Paper Test and Multiple Assessment(5+5)	10 Marks
Portfolio	05 Marks
Lab Practical (Lab activities to be done from the prescribed books)	05 Marks

TOPIC WISE ANALYSIS OF EXAMPLES AND QUESTIONS

NCERT TEXT BOOK

Sl. No.	Topic	Number of Questions for revision		
		From solved examples	Questions from exercise	Total No of Questions
1	Real Number	11	18	29
2	Polynomials	09	08	17
3	Pair of linear equations in two variables	19	21	40
4	Quadratic Equation	18	24	42
5	Arithmetic Progression	16	44	60
6	Triangle	14	55	69
7	Coordinate Geometry	15	25	40
8	Introduction to trigonometry	15	27	42
9	Some Applications of Trigonometry	7	16	23
10	Circle	3	17	20
11	Construction	2	14	16
12	Area Related to Circle	6	35	41
13	Surface Area and Volume	14	31	45
14	Statistics	09	25	34
15	Probability	13	25	38
	Total	**171**	**385**	**556**

DETAILS OF THE CONCEPTS TO BE MASTERED BY EVERY CHILD OF CLASS X WITH EXERCISE AND EXAMPLES OF NCERT TEXT BOOKS.

SYMBOLS USED

TG/LG is idea identified error analysis of answers of Q.P of last three year.

* - Important Question   a - Low   T.G-Teaching Gap
** -Very Important Question   b - Average   L.G-Learning Gap
*** -Very Very Important Question   c - Higher

SI. No	TOPIC	CONCEPTS	DEGREE OF IMPORTAN	TG/LG	DEGREE	REFERENCES(NCERT BOOK)
01	Real Number	Euclid's division Lemma & Algorithm	***	L.G	a	Example -1,2,3,4 Ex:1.1 Q:1,2,4
		Fundamental Theorem of Arithmetic	***	L.G	a	Example -5,7,8 Ex:1.2 Q:4,5
		Revisiting Irrational Numbers	***	L.G	b	Example -9,10,11 Ex: 1.3 Q:1.2 Th:1.4
		Revisiting Rational Number and their decimal Expansion	**	L.G	a	Ex -1.4 Q:1
02	Polynomials	Meaning of the zero of Polynomial	*	L.G	a	Ex -2.1 Q:1
		Relationship between zeroes and coefficients of a polynomial	**	L.G	a	Example -2,3 Ex-2.2 Q:1
		Forming a quadratic polynomial	**	L.G	b	Ex -2.2 Q:2
		Division algorithm for a polynomial	*	L.G	b	Ex -2.3 Q:1,2
		Finding the zeroes of a polynomial	***	L.G	a	Example: 9 Ex -2.3 Q:1,2,3,4,5 Ex-2.4,3,4,5
03	Pair of Linear Equations in two variables	Graphical algebraic representation	*	L.G	b	Example:2,3 Ex -3.4 Q:1,3
		Consistency of pair oflinerequations	**	L.G	a	Ex -3.2 Q:2,4
		Graphical method of solution	***	L.G	b	Example: 4,5 Ex -3.2 Q:7
		Algebraic methods of solution Substitution method	**	L.G	b	Ex -3.3 Q:1,3
		Elimination method				Example-13 Ex:3.4 Q:1,2 Example-15,16 Ex:3.5 Q:1,2,4
		Cross multiplication method				Example-19 Ex-3.6 Q:1(ii),(viii),2 (ii),(iii)
		Equation L.Greducible to pair of liner equation in two variable				

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04	Quadratic Equation	Standard form of quadratic equation	***	L.G	a	NCERT Text book Q.1.2, Ex 4.1
		Solution of quadratic equation by factorization	***	L.G	a	Example3,4,5,Q.1,5Ex.4.2
		Solution of quadratic equation by completing the square	**	L.G	b	Example 8,9 Q.1 Ex. 4.3
		Solution of quadratic equation by quadratic formula	***	L.G	a	Example. 10,11,13,14,15 , Q2,3(ii)
		Nature of roots	***	L.G	a	Example 16 Q.1.2, Ex. 4.4
05	Arithmetic progression	General form of an A.P.	*	L.G	a	Exp-1,2,Ex.5.1Q.s2(a), 3(a),4(v)
		nth term of an A.P.	***	L.G	a	Exp. 3,7,8 Ex. 5.2 Q.4,7,11,16,17,18
		Sum of first n terms of an A.P.	***	L.G	b	Exp.11,13,15 Ex. 5.3, Q.No.1(i, ii)
06	Triangles	1) Similarity of Triangles	***	T.G	C	Theo:6.1 Example:1,2,3 Ex:6.2 Q:2,4,6,9,10
		2) Criteria for Similarity of Triangles	**	T.G	C	Example:6,7 Ex:6.3 Q:4,5,6,10,13,16
		3) Area of Similar Triangles	***	L.G	B	Example:9 The:6.6 Ex:6.4 Q:3,5,6,7
		4) Pythagoras Theorem	***	L.G	b	Theo:6.8 & 6.9 Example:10,12,14, Ex:6.5 Q:4,5,6,7,13,14,15,16
07	Coordinate geometry	Distance formula	**	L.G	b	Exercise7.1,Q.No1,2,3,4,7,8
		Section formula Midpoint formula	**			Example No. 6,7,9 Exercise7.2,Q.No.1,2,4,5 Example 10.
		Area of Triangle	** ***	L.G	a	Ex.1,2,14 Ex 7.3 QNo-12,4 Ex.7.4, Q.no.- 2
08	Introduction to Trigonometry	1) Trigonometric Ratios	*	L.G	a	Ex:8.1 Q:1,2,3,6,8,10
		2)Trigonometric ratios of some specific angles	**	L.G	b	Example:10,11 Ex:8.2 Q:1,3
		3) Trigonometric ratios of complementary angles	**	L.G	a	Example:14,15 Ex:8.3 Q:2,3,4,6
		4) Trigonometric Identities	***	L.G	b	Ex:8.4 Q:5(iii,v,viii)
09	Some application of Trigonometry	Heights and distances	**	L.G	b	Example-2,3,4 Ex 9.1 Q 2,5,10,12,13,14,15,16
10	Circles	Tangents to a circle	***	L.G	a	Q3(Ex10.1) Q 1,Q6,Q7(Ex 10.2),4
		Number of tangents from a point to a circle	***	L.G	a	Theorem 10.1,10.2 Eg 2.1 Q8,9,,10,12,13(Ex 10.2)

11	Constructions	Division of line segment in the given Ratio	*	L.G	b	Const 11.1 Ex 11.1 Q.no. 1
		Construction of triangle similar To given triangle as per given scale	***	L.G	b	Ex 11.1 Qno-2,4,5,7
		Construction of tangents to a Circle	***	L.G	a	Ex 11.2 Q.no. 1,4
12	Area related to circles	Circumference of a circle	*	L.G	a	Example 1 Exercise 12.1 No. 1,2,4
		Area of a circle	*	L.G	a	Example 5,3
		Length of an arc of a circle	*	L.G	a	Exercise 12.2 Q No 5
		Area of sector of a circle	**	L.G	b	Example 2 Exercise 12.2 No 1.2
		Area of segment of a circle	**	L.G	a	Exercise 12.2 Qno 4,7,9,3
13	Surface area and volumes	Combination of figures	***	L.G	b	Ex 12.3 Example 4.5 1,4,6,7,9,12,15
		Surface area of a combination of solids	**	T.G	c	Example 1,2,3 Exercise 13.1 Q1,3,6,7,8
		Volume of combination of a solid	**	L.G	b	Example 6 Exercise 13.2
		Conversion of solids from one shape to	***	L.G	a	Example 8 & 10 Exercise 13.3
		Frustum of a cone	***	L.G	b	Example 12& 14 Exercise 13.4
14	Statistics	CONCEPT 1 Mean of grouped data	***	L.G	a	
		1. Direct Method	*	L.G	b	Example:2 Ex:14.1 Q:1&3
		2. Assumed Mean Method	*	L.G	b	Ex:14.1 Q:6
		3. Step Deviation Method		L.G	b	Ex:14.1 Q:9
		CONCEPT 2	***	L.G		
		Mode of grouped data		L.G	a	Example:5 Ex:14.2 Q:1,5
		CONCEPT 3	***	L.G		
		Median of grouped data		L.G	a	Example:7,8 Ex:14.3 Q1,3,5
		CONCEPT 4	**	L.G		
		Graphical representation of c.f.(give)	***	L.G	b	Example:9 Ex:14.4 Q:1,2,3
15	Probability	Events	*	L.G	a	Ex 15.1 Q4,8,9
		Probability lies between 0 and1	**	L.G	b	Exp- 1,2,4,6,13
		Performing experiment	***	L.G	a	Ex 15 1,13,15,18,24

Real Numbers

(Key Points)

Real

Numbers

```
                    Real Numbers
            ┌───────────────┴───────────────┐
            ▼                               ▼
    ┌──────────────────┐          ┌──────────────────┐
    │ RATIONAL         │          │ IRRATIONAL       │
    │ NUMBERS (Q)      │          │ NUMBERS (I)      │
    └──────────────────┘          └──────────────────┘
```

NATURAL NUMBERS(COUNTING NO 1,2,3,...	**WHOLE NUMBERS (W) (0,1,2,3,4,...)**	**INTEGER (Z)**

NEGATIVE INTEGERS (-1,-2,-3,)	**ZERO (0)**	**POSITIVE INTEGERS (1, 2, 3...)**

DECIMAL FORM OF REAL NUMBERS

TERMINATING DECIMAL (2/5,¾,....) (RATIONAL NUMBERS)	**REPEATING DECIMAL (RECURRING DECIMAL) (1/3, 2/7,3/11,...) (RATIONAL NUMBERS)**	**NON TERMINATING NON RECURRING (1.010010001...) (IRRATIONAL NUMBERS)**

1. **Euclid's Division lemma:-G**iven Positive integers a and b there exist unique integer's q and r satisfying   a=bq+r, where 0≤r<b, where a,b,q and r are respectively called as dividend, divisor, quotient and remainder.

2. **Euclid's division Algorithm:-**To obtain the HCF of two positive integers say c and d, with c>d, follow the steps below:

   Step I: Apply Euclid's division lemma, to c and d, so we find whole numbers, q and n r such that c=dq +r, $0 \leqslant r < d$
   Step II: If r =0, d is the HCF of c and d. If r ≠ 0, apply the division lemma to d and r.
   Step III: Continue the process still the remainder is zero. The divisor at this stage will be the required HCF
    Note: - Let a and b be positive integers .If a=b q +r, 0≤r<b, then HCF (a, b) = HCF (b, r)

3. **The Fundamental theorem of Arithmetic:-**
   Every composite number can be expressed (factorized) as a product of primes and this factorization is unique, apart from the order in which the prime factors occur.
   Ex.: 24= 2 x 2 x 2 x 3 = 3 x 2 x 2 x 2

Theorem: Let x be a rational number whose decimal expansion terminates. Then x can be expressed in the form of p/q where $p \ and \ q$ are co-prime and the prime factorization of q is of the form of $2^n x 5^m$, where n,m are non-negative integers.

Ex:-

$$\frac{7}{10} = \frac{7}{2 \times 5} = 0.7$$

Theorem: $x = \dfrac{p}{q}$ Let x be a rational number such that the prime factorization of q is not of the form of $2^n x \ 5^m$, where n,m are non-negative integers. Then x has a decimal expansion which is non-terminating repeating (recurring).

Ex:-

$$\frac{7}{6} = \frac{7}{2 \times 3} = 1.1666\ldots\ldots.$$

Theorem: For any two positive integers a and b,
HCF (a,b) XLCM (a,b) =aXb

Ex.:4&6; HCF (4, 6) =2, LCM (4,6) =12; HCFXLCM=2X12=24

Ans.: a x b=24

LEVEL-I

1. Write a rational number between $\sqrt{2} \ and \sqrt{3}$

2. The decimal expansion of the rational no. $\dfrac{43}{2^4 5^3}$ will terminate after how many places of decimal?

3. Find the (HCF x LCM) for the numbers 100 and 190

4. If $\dfrac{p}{q}$ is a rational number (q ≠ 0). What is the condition on q so that the decimal representation of $\dfrac{p}{q}$ is terminating?

5. State whether the number ($\sqrt{2} - \sqrt{3}$)($\sqrt{2} + \sqrt{3}$) is rational or irrational justify.

6. Write one rational and one irrational number lying between 0.25 and 0.32.

7. Express107 in the form of 4q+3 for some positive integer q.

8. Write whether the rational number $\frac{51}{1500}$ will have at erminating decimal expansion or a non terminating repeating decimal expansion.

9. Show that any positive odd integer is of the form 6q+1 or 6q+3 or 6q+5, where q is some integer.

10. Express 0.2545454…………..as a fraction in simplest form.

LEVEL-II

1. Use Euclid's division algorithm to find the HCF of 1288 and 575.
2. Check whether 5x3x11+11 and 5x7+7X3 are composite number and justify.
3. Check whether 6^n can end with the digit 0, where n is any natural number.
4. Given that LCM (26,169) =338, write HCF (26,169).
5. Find the HCF and LCM of 6,72 and 120 using the prime factorization method.
6. Use Euclid's division lemma to show that the square of any positive integer is either of the form 3m or 3m+1for some integer m.
7. Use Euclid's division lemma to show that the cube of any positive integer is of the form 9m, 9m+1 or 9m+8 for some integer m.

LEVEL-III

1. Show that √3 is an irrational number.
2. Show that 5+3 $\sqrt{2}$ is an irrational number
3. Show that square of an odd positive integer is of the form 8m+1, for some integer m.
4. Find the LCM & HCF of 26 and 91 and verify that HCF x LCM=product of the two numbers
5. Prove that $\sqrt{7}$ is irrational
6. Show that one and only one out of n, n+2, n+4 is divisible by 3, where n is any positive integer.
7. Find the HCF of 65&117 and express it in the form of 65m+117n.

(PROBLEMS FOR SELF EVALUATION/HOTS)

1. State the fundamental theorem of Arithmetic.
2. Express 2658 as a product of its prime factors.
3. Find the LCM and HCF of 17, 23 and 29.
4. Prove that $\sqrt{2}$ is not a rational number.
5. Find the largest positive integer that will divide 122,150 and 115 leaving remainder 5,7and11 respectively.
6. Show that there is no positive integer n for which $\sqrt{n-1} + \sqrt{n+1}$ is rational.
7. Using prime factorization method, find the HCF and LCM of 72,126 and 168. Also show that HCF X LCM ≠ product of three numbers.
8. Three sets of English, Mathematics and Science books containing 336, 240 and 96 books respectively have to be stacked in such away that all the books are stored subject wise and the height of each stack is the same. How many stacks will be there?

9. A person wanted to distribute 96 apples and 112 oranges among poor children in an orphanage. He packed all the fruits in boxes in such away that each box contains fruits of the same variety, and also every box contains an equal number of fruits.

- Find the maximum number of boxes in which all the fruits can be packed.
- Which concept have you used to find it?

10. Teacher draws the factor tree, given in figure and ask the students to find the value

of x without finding the value of y and z. Shaurya gives the answer x=136

a) Is his answer correct?
b) Give reason for your answer.
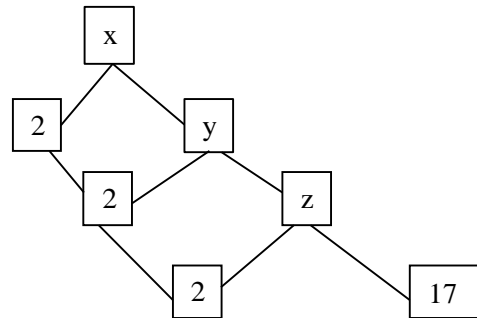
Answer Level-I

1. 1.42 (flexible)
2. After 4 places of decimal.
3. 19000
4. q is of the form $2^n.5^m$, where m and n are non-negative integers.
5. Rational number
6. One rational number=26/100, one irrational no. =0.27010010001.........
7. 4 X26+3
8. Terminating

10.14/55

Level-II

1. 23
2. Composite number and justification show that numbers are having more than two factors
3. No,6^n cannot end with the digit 0.
4. 13
5. HCF=6, LCM = 360

6. ncert book.ex1.1/q.n.4

7. ncert book ex1.1/q.n.5

16

LEVEL-III

      a. See
          textbook. ex
          9 p.no 13

2. See textbook ex 1.3
q.no2

3. See textbook

4. LCM = 182   HCF=13
Product =2366

5   See textbook

7. HCF =13

Problems for self-evaluation/Hots

1. Write statement of fundamental theorem

2. 2658= 2x3x443

3. HCF= 1 LCM =11339

4. Refer Ncert book

5. Number =13

7. HCF = 6 LCM = 504 NOT EQUAL

8. Number of books in a stack = 48 (i) No.of boxes=14

9.16 fruits in a box Number of Boxes =13

   Number System &HCF
10. (a) Yes, his answer is correct.
     (b) Z =2 X 17 = 34, Y = 2 X 34 = 68, X = 2 x 68 = 136
    (c) Knowledge of prime factorization.

Polynomial

Polynomial

An expression of the form p(x) = $a_0 + a_1x + a_2x^2 + \cdots \ldots \ldots \ldots \ldots +a_nx^n$ where $a_n \neq 0$ is called a polynomial in one variable x of degree n, where; $a_0, a_1, a_2 \ldots \ldots \ldots \ldots \ldots.. a_n$ are constants and they are called the coefficients of $x_0, x, x^2 \ldots \ldots \ldots x^n$. Each power of x is a non-negative integer.

Eg: $-2x^2 - 5x + 1$ is a polynomial of degree 2

Note: $\sqrt{x} + 3$ is not a polynomial

- A polynomial p$(x) = ax + b$ of degree 1 is called a linear polynomial Eg: $5x - 3, 2x$ etc
- A polynomial p$(x) = ax^2 + bx + c$ of degree 2 is called a quadratic polynomial Eg: $2x^2 + x - 1$
- A polynomial $p(x) = ax^3 + bx^2 + cx + d$ of degree 3 is called a cubic polynomial. Eg: $\sqrt{3}x^3 - x + \sqrt{5}, x^3 - 1$ etc

Zeroes of a polynomial: A real number k is called a zero of polynomial p(x) if p(k)=0. If the graph of y= p(x) intersects the X-axis at n times, the number of zeroes of y= p(x) is n.

- A linear polynomial has only one zero.
- A quadratic polynomial has two zeroes.
- A cubic polynomial has three zeroes.

Graphs of different types of polynomials:

- Linear polynomial:- The graph of a linear polynomial ax+b is a straight line, intersecting X- axis at one point
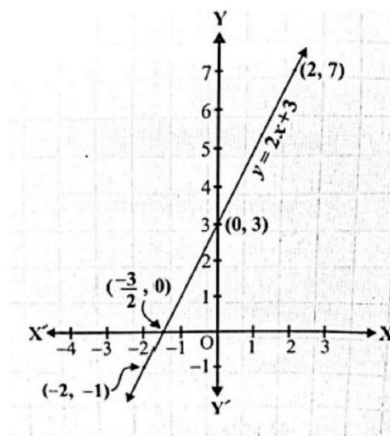
- Quadratic polynomial:-

   (i) Graph of a quadratic polynomial $p(x) = ax^2 + bx + c$ is a parabola open upwards like U, if a>0 & intersects x-axis at maximum two distinct points.
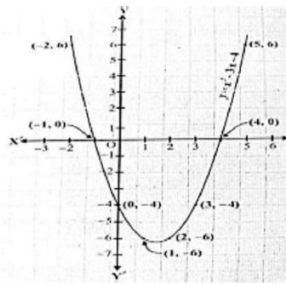
(ii) Graph of a quadratic polynomial p(x)=$ax^2 + bx + c$ is a parabola open downwards like ∩ if a<0 & intersects x-axis at maximum two distinct points
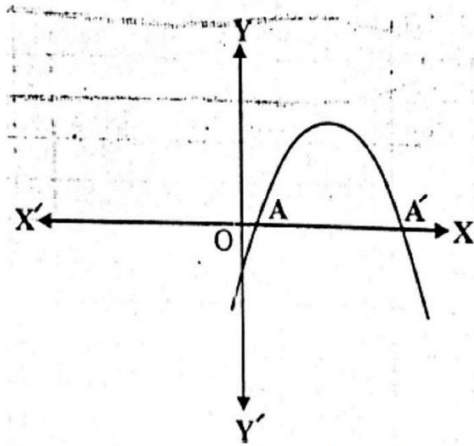
- Cubic polynomial and its graph:- in general a polynomial p(x) of degree n crosses the x axis at most n points.
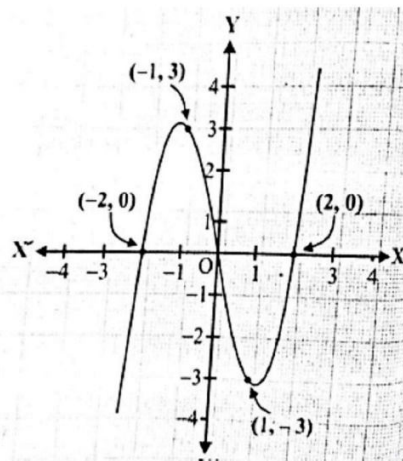
For a quadratic polynomial:- If α, β are zeroes of $p(x) = ax^2 + bx + c$ then,

1. Sum of zeroes = $\alpha + \beta = -\dfrac{b}{a} = \dfrac{-coefficients\ of\ x}{coefficient\ of\ x^2}$

2. Product of zeroes= $\alpha.\beta = \dfrac{c}{a} = \dfrac{constant\ term}{coefficient\ of x^2}$

- A quadratic polynomial whose zeroes are $\alpha\ and\ \beta$, is given by:

$$p(x) = x^2 - (\alpha + \beta)x + \alpha\beta$$

- If $\alpha, \beta\ and\ \gamma$ are zeroes of the cubic polynomial $ax^3 + bx^2 + cx + d$ then:

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$$\alpha + \beta + \gamma = -\frac{b}{a}$$
$$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}$$
$$\alpha\beta\gamma = \frac{-d}{a}$$

- If $\alpha, \beta \& \gamma$ are zeroes of a cubic polynomial p(x),

$$p(x) = x^3 - (\alpha + \beta + \gamma)x^2 + (\alpha\beta + \beta\gamma + \gamma\alpha)x - \alpha\beta\gamma$$

Division algorithm for polynomials: If p(x) and g(x) are any two polynomials with g(x)$\neq$ 0, then we have polynomials q(x) and r(x) such that
P(x) = g(x)$\times q(x) + r(x)$, where r(x) =0 or degree of r(x) <degree of g(x).

Nature of graph of polynomial$P(x) = ax^2 + bx + c$:-
Case-1 When polynomial $ax^2 + bx + c$ is factorable in two distinct linear factors.

In this case, curve cuts X- axis at two distinct points. The co-ordinate of the vertex of parabola are(-b/2a, -D/2a) where D=$b^2 - 4ac$. The x co-ordinates of these points are the two zeroes of the polynomial.
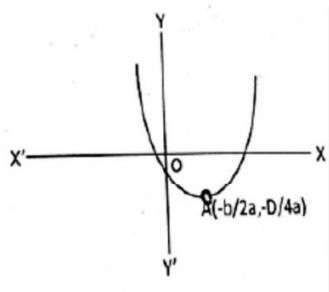
(i) a>0
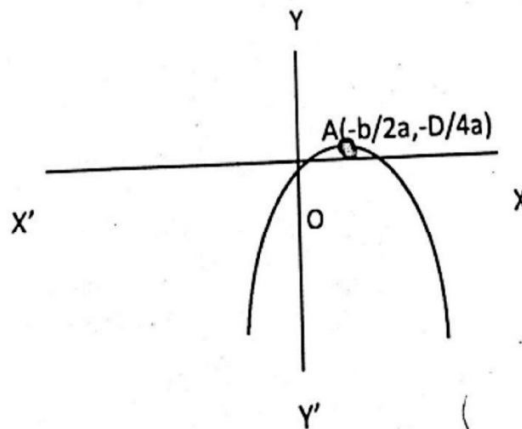
(ii) a<0

Case 2:- When Polynomial $ax^2 + bx + c$ is factorisable into two equal factors.

In this case, curve touches X-axis at the point (-b/2a, 0). The x- Co-ordinates of the point gives two equal zeroes of the polynomial.

(i)   a>0   (ii) a<0
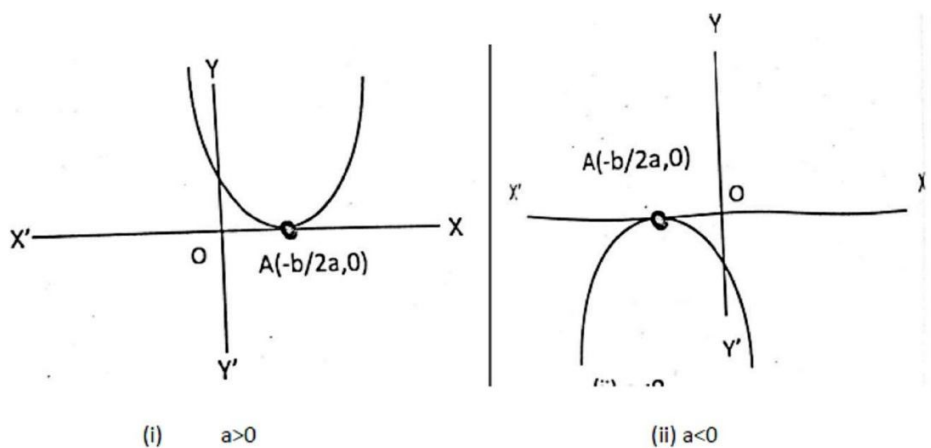
Case- 3 When Polynomial $ax^2 + bx + c$ is not factorizable. In this case, the curve doesn't cut or touches X-axis

(ii) a<0
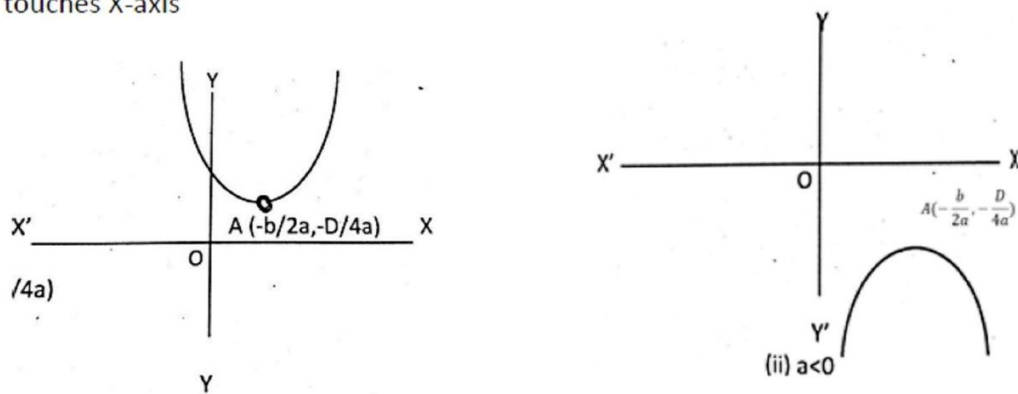
Level –I

1. Find the value of zeroes of the polynomials p(x) as shown in the graph and hence find the polynomial.(CBSE 2014-15).
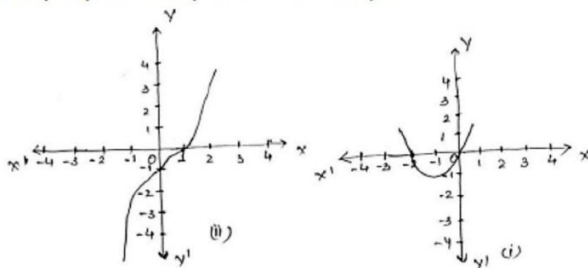
2. Let α and β are the zeroes of a quadratic polynomial $2x^2 - 5x - 6$ then form a quadratic polynomial whose zeroes are $\alpha + \beta \ and \ \alpha\beta$. (CBSE 2011)

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8. Check whether $x^2 + 3x + 1$ is a factor of $3x^4 + 5x^3 - 7x^2 + 2x + 2$?

9. Can x-7 be the remainder on division of a polynomial P(x) by 7x+2? Justify your answer.

10. What must be subtracted from the polynomial $f(x) = x^4 + 2x^3 - 13x^2 - 12x + 21$ s0 that the resulting polynomial is exactly divisible by $x^2 - 4x + 3$.

11. Write the degree of the zero polynomial.

12. Find the zeros of the quadratic polynomial $6x^2 - 7x - 3$ and verify the relationship between the zeros and the coefficients.

13. Find the quadratic polynomial sum of whose zeros is $2\sqrt{3}$ and their product is 2.

Level II

9. If the sum of squares of zeros of the polynomial $6x^2 + x + k$ is 25/36 find the value of k.

10. If one zero of the quadratic polynomial $f(x) = 4x^2 - 8kx - 9$ is negative of the other, find the value of k.

11. Find the value of k for which the quadratic polynomial $9x^2 - 3kx + k$ has equal zeros

12. On dividing $3x^3 - 2x^2 + 5x - 5$ by the polynomial p(x), the quotient and remainder are $x^2 - x + 2$ and -7 respectively. Find p(x)

13. Find all the zeros of polynomial $x^4 + x^3 - 9x^2 - 3x + 18$ if the two of its zeros are $\sqrt{3}$ and $-\sqrt{3}$.

14. If α and β are the zeros of quadratic polynomial $p(x) = x^2 - (k-6)x + (2k+1)$
    Find the value of k if $α + \beta = αβ$.

15. If the zeros of polynomial x2-5x+k are the reciprocal of the zeros then find the value of k.

16. If α and β are the zeros of the quadratic polynomial x2 -6x +a find the value of 'a'
    If 3α +2β=20

17. On dividing 3x3+4x2+5x-13by a polynomial g(x) ,the quotient and remainder are 3x+10and 16x-43 respectively. Find the polynomial g(x).

18. If α and β are the zeroes of a polynomial x2-x-30, then form a quadratic Polynomial whose zeroes are 2-α and 2- β

19. If α, β and γ are zeroes of the polynomial $6x^3 + 3x^2 - 5x + 1$, then find the values of $\alpha^{-1} + \beta^{-1} + \gamma^{-1}$. (CBSE 2010)

20. Form a cubic polynomial whose zeroes are 3, 2 and -1. Hence find
    (i)   Sum of its zeroes
    (ii) Sum of the product, taken two at a time
    (iii) Product of its zero.

(SELF EVALUATION QUESTIONS)

21. Find the number of zeroes of p(x) in each case, for some polynomials p(x).
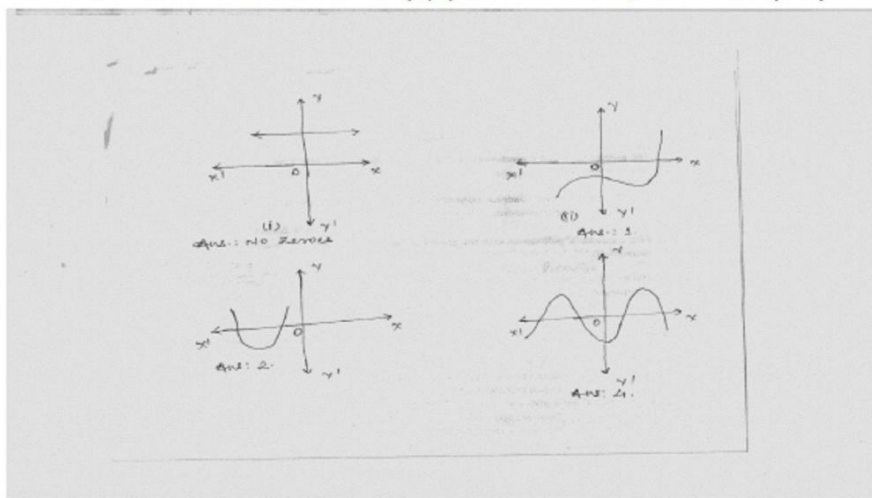
22. If α and β are the zeroes of the equation $6x^2 + x - 2 = 0$, find $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$

23. If one of the zeroes of the polynomial $2x^2 + px + 4 = 0$ is 2, find the other zero, also find the value of p

24. If one zero of the polynomial $(a^2 + 9)x^2 + 13x + 6a$ is reciprocal of the other. Find the value of a. (All India)

25. If α be the number of person who take junk food, β be the person who take food at home and α and β be the zeroes of quadratic polynomial $f(x) = x^2 - 3x + 2$, then find a quadratic polynomial whose zeroes are $\frac{1}{2\alpha+\beta}$ and $\frac{1}{2\beta+\alpha}$, which way of taking food you prefer and why?

26. If the number of apples and mangoes are the zeroes of polynomial $3x^2-8x-2k+1$. and the number of apples are 7 times the number of mangoes, then find the number of zeroes and value of k.

ANSWERS

LEVEL I

1. A) 1 polynomial x-1
   B) -2, 0 polynomial x^2+2x

2. $K(2x^2 +x-15)$

3. Check by division method.

4. No Degree of remainder should be less than Degree of divisor

5. 2x-3 should be subtracted.

6. 0.

7. 3/2 and -1/3 are the zeroes Verification to be done

8. $x^2-2\sqrt{3}x+2$

LEVEL II

9. k= -2
10. k= 0
11. k= 0 or k=4
12. 3x-1
13. Other Zeroes 2 and -3
14. K= -7
15. K=1
16. a= -16

LEVEL III

17. $g(x) =x^2-2x+3$
18. $x^2-3x-28$
19. 5
20.   (i) 4
    (ii) 1
    (iii). -6

SELF EVALUATION QUESTIONS

21. (i) no zeroes
  (ii) 1
   (iii). 2
   (iv). 4

22. -2
23. Other zero 1 and P= -6
24. a=3
25. $20x^2-9x+1$
26. 2 zeroes, K= -2/3

<u>Pair of Linear Equations in Two Variables</u>

<u>(Key Points)</u>

- An equation of the form ax+by+c=0, where a,b,c are real nos. and $a^2+b^2 \neq 0$ is called a linear equation in two variables x and y.

  Ex: (i) x+5y+2=0   (ii)$\frac{3}{2}$ x - y = 1

- The general form for a pair of linear equations in two variables x and y is

  $a_1x+b_1y+c_1=0$

  $a_2x + b_2y + c_2 = 0$

  Where a_1,b_1,c_1,a_2,b_2,c_2 are all real numbers and $a_1 \neq 0, b_1 \neq 0, a_2 \neq 0, b_2 \neq 0$. Example

  x + 3y − 6 = 0

  2x − 3y − 12=0

- Graphical representation of a pair of linear equations in two variables:

  $a_1x+b_1y+c_1=0$

  $a_2x + b_2y + c_2 = 0$

  (i) Will represent intersecting lines if

  $$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

  I.e. unique solution. And these types of equations are called consistent pair of linear equations.

  Ex: x − 2y = 0

      3x + 4y − 20 = 0

  Co-ordinates of the point of intersection gives the solution of the equations.
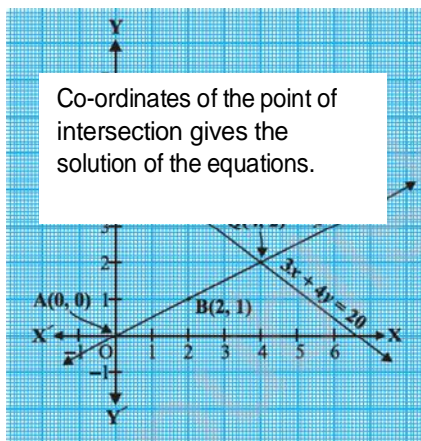
  (ii) Will re present overlapping or coincident lines if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

  i.e. infinitely many solutions, consistent or dependent pair of linear equations

  Ex:2x+3y−9=0

      4x+6y−18=0
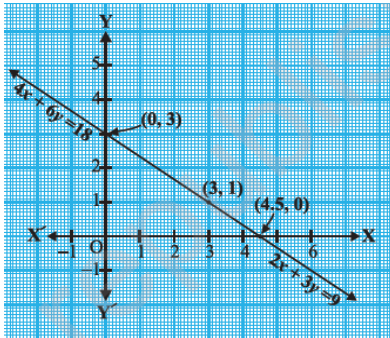
The graph is Coincident lines,

(iii) will represent parallel lines if $\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} \neq \dfrac{c_1}{c_2}$

i.e.no solution and called inconsistent pair of linear equations. Ex:

x + 2y − 4 =0

2x + 4y − 12 = 0
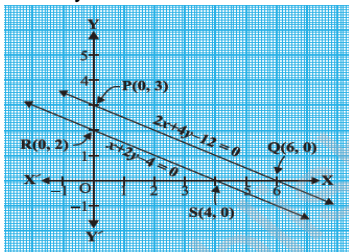
Parallel lines, no solution.

- Algebraic methods of solving a pair of linear equations:

(i)   Substitution method
(ii)   Elimination Method
(iii)   Cross multiplication method

Level -I

1. Find the value of 'a' so that the point (2,9) lies on the line represented by ax-3y=5

2. Find the value of k so that the lines 2x–3y = 9 and kx - 9y = 18 will be parallel.

3. Find the value of k for which x + 2y = 5,3x + ky+15 = 0 is inconsistent

4. Check whether given pair of lines is consistent or not 5x–1=2y,y=$\dfrac{-1}{2}$ + $\dfrac{5}{2}$ x

5. Determine the value of 'a' if the system of linear equations 3x+2y-4=0and ax–y–3=0 will represent intersecting lines.

6. Write any one equation of the line which is parallel to√2x–√3y=5

7. Find the point of intersection of line -3x+7y=3 with x-axis

8. For what value of k the following pair has infinite number of solutions.

    (k-3)x+3y=k

    K (x + y) =12

9. Write the condition so that $a_1x + b_1y = c_1$ and $a_2x + b_2y = c_2$ have unique solution.

Level -II

1. 5 pencils and 7 pens together cost Rs.50 whereas 7 pencils and 5 pens together cost Rs.46.Find the cost of one pencil and that of one pen.

2. Solve the pair of linear equation:
   $3x - y = 3$
   $7x + 2y = 20$

3. Find the fraction which becomes to 2/3 when the numerator is increased by 2 and equal to 4/7 when the denominator is increased by 4.

4. Solve the pair of linear equation:

   $px + qy = p - q$

   $q x - py = p + q$

5. Solve the equation using the method of substitution:

   $3x - 5y = -1$
   $x - y = -1$

6. Solve the equations:

   $$\frac{1}{2x} - \frac{1}{y} = -1$$

   $$\frac{1}{x} + \frac{1}{2y} = 8 ;$$ Where, $x \neq 0$, $y \neq 0$

7. Solve the equations by using the method of cross multiplication:

   $x - y = 7$

   $5x + 12y = 7$

Level -III

1. Draw the graph of the equations

   $4x - y = 4$

   $4x + y = 12$
   Determine the vertices of the triangle formed by the lines representing these equations and the x-axis.

   Shade the triangular region so formed

2. Solve Graphically
   $x - y = -1$
   $3x + 2y = 12.$
   Calculate the area bounded by these lines and the x- axis.

3. Solve for u& v.

   4u–v=14uv
   3u + 2v = 16uv where u≠0, v≠ 0

4. Ritu can row downstream 20 km in 2 hours, and upstream 4km in 2hours. Find her speed of rowing in still water and the speed of the current. (HOTS)

5. In a $\triangle ABC, \angle C = 3\angle B = 2(\angle A + \angle B)$ find the these angle. (HOTS)

6. 8 men and 12 boys can finish a piece of work in 10 days while 6 men and 8 boys can finish it in14 Days.

    Find the time taken by 1 man alone and that by one boy alone to finish the work. (HOTS)

7. Find the value of K for which the system of linear equations 2x + 5y = 3, (k+1)x + 2(k+2)y = 2 K will have infinite number of solutions. (HOTS)

SELF EVALUTION

1. Solve for x and y:

    $x + y = a + b$

    $ax - by = a^2 - b^2$

2. For what value of k will the equation x +5y − 7 = 0 and 4x + 20y + k = 0 represent coincident lines?

3. Solve graphically:

    3x + y + 1 = 0

    2x - 3y +8 = 0

4. The sum of digits of a two digit number is 9.If 27 is subtracted from the number, the digits are reversed. Find the number.

5. Draw the graph of x+2y−7=0and 2x−y − 4 =0. Shade the area bounded by these lines and Y-axis.

6. Students of a class are made to stand in rows. If one student is extra in a row, there would be 2rows less. If one student is less in a row there would be 3 rows more. Find the number of the students in the class.

7. Aman travels 370 km partly by train and remaining by car. If he covers 250 km by train and the rest by the car it takes him 4hours, but if he travels 130 km by train and th rest by car, he takes18minutes longer. Find the speed of the train and that of the car.

8. Given linear equation 2x+3y − 8 = 0, write an other linear equation such that the geometrical representation of the pairs of lines are (i) intersecting lines, (ii) Parallel Lines.

9. Solve for x and y.

$(a-b)x + (a+b)y = a^2 - 2ab - b^2$

$(a+b)(x+y) = a^2 + b^2$ **(CBSE 2004, '07C,'08)**

10. The sum of two numbers is 8 and the sum of the irreciprocalis 8/15.Find the numbers.

(CBSE 2009)

11. The owner of a taxi cab company decides to run all the cars he has on CNG fuel instead
Of petrol/diesel. The car hire charges in city comprises of fixed charges together with the charge for the
Distance covered. For a journey of 12km, the charge paid Rs.89 and for a journey of 20km, the charge paid is Rs.145.
    i.   What will a person have to pay for travelling a distance of 30km?
    ii. Which concept has been used to find it?
    iii. Which values of the owner have been depicted here?

12. Riya decides to use public transport to cover a distance of 300km.She travels this distance partly by train and remaining by bus. She takes 4 hours if she travels 60 km by bus and the remaining by train. If she travels100kmby bus and the remaining by train, she takes 10minutes more.
    i.   Finds peed of train and bus separately.
    ii. Which concept has been used to solve the above problem?
    iii. Which values of Riya have been depicted here?

ANSWER

LEVEL-I

Q1 a=16

Q2. k=6

Q3 .k=6

Q4. Consistent

Q5. a ≠ 3/2

Q6.$5\sqrt{2}x - 5\sqrt{3}y = 5\sqrt{5}$(May be another solution also)

Q7. (-1,0)

Q8. k= 6

Q9. $\dfrac{a_1}{a_2} \neq \dfrac{b_1}{b_2}$

LEVEL-II

Q1.   Cost of one pencil=Rs.3

        Cost of one pen=Rs.5

Q2.   X = 2, y = 3

Q3. 28/45

Q4.   x=1, y = -1

Q5. $x = -2, \ y = -1$

Q6.$x = \dfrac{1}{6}, y -\dfrac{1}{4}$

Q7.$x = 11, y = -4$

LEVEL-III

Q1. (2, 4)(1,0)(3,0)

Q2. x = 2, y = 3 and area = 7.5 unit ^{2}

Q3. u = ½ , v = ¼

Q4.Speed of the rowing in still water = 6km/hr

    Speed of the current=4km/hr.

Q5.∠A=20^0,∠B=40^0,∠C=120^0.

Q6: One man can finish work in140days. One boy can finish work in 280days.

Q7. K = 3

SELF EVALUATION

Q1. X=a y=b

Q2.K=-28

Q3.X= -1, y=2

Q4.   63

Q6. 60

Q7.Speed of the train=100km/h, speed of the car=80km/h

Q8.(i)4x-3y-8=0(may be another equation also)

(ii) 4x+6y+16 =0 (may be another equation also)

Q9. X= a+b, y= - 2ab/(a+b)

Q10. 3,5

Q11. (i) Rs.215,

(ii) A pair of linear equations in two variables has been used to find it.

(iii) Awareness of environment.

Q12. (i) The speed of the train = 80 km/h, the speed of the bus = 60km/h

(ii) A pair of linear equations in two variables has been used.

(iii) Controlling the pollution of the environment.

QUADRATIC EQUATIONS

KEY POINTS

1. The general form of a quadratic equation is $ax^2+bx+c=0$, $a\neq o$.a,b and c are real numbers.

2. A real number α is said to be a root of quadratic equation $ax^2+bx+c=0$ where $a\neq0$ if $a\alpha^2+b\alpha+c=0$.The zeroes of the quadratic polynomial ax^2+bx+c and the roots of the corresponding quadratic equation $a x^2 + b x + c = 0$ are the same.

3. Discriminant:-The expression $b^2 - 4ac$ is called Discriminant of the equation $ax^2+bx+c = 0$ and is usually denoted by D.Thus Discriminant $D = b^2 - 4ac$.

4. Every quadratic equation has at most two roots which may be real, coincident or no real roots.

5. IF α and β are the roots of the equation $ax^2+bx+c=0$ then

$$\alpha = \frac{-b+\sqrt{b^2-4ac}}{2a} \qquad \beta = \frac{-b-\sqrt{b^2-4ac}}{2a}$$

6. Sum of the roots $, \alpha + \beta = -\frac{b}{a}$

   Product of roots $\alpha\beta = \frac{c}{a}$

7. Forming quadratic equation, when the roots α and β are given by

$$x^2 - (\alpha + \beta)x + \alpha.\beta = 0$$

8. Nature of roots of $ax^2+bx+c=0$
   a. If $D > 0$, then roots are real and unequal.
   b. If $D = 0$,then the equation has equal and real roots.
   c. If $D < 0$,then the equation has no real roots.
   d. If $D > 0$ and D is a perfect square, then roots are rational and unequal.
   e. If $D > 0$ and D is not a perfect square then roots are irrational and unequal.

9. Irrational roots always occur in conjugate pairs. If $2+\sqrt3$ is one of the roots of the quadratic equation then other root is $2-\sqrt3$.

10. If $a.b>0$ the na>0 and $b>0$ or $a<0$ and $b<0$ If $a.b<0$ the na>0 and $b<0$ or $a<0$ and $b>0$.

LEVEL-I

1. If $\frac{1}{2}$ is a root of the equation $x^2 + kx - \frac{5}{4}= 0$, then find the value of K.

2. If $D > 0$,then write the roots of a quadratic equation $ax^2+bx+c = 0$

3. Find the Discriminant of $x^2+ 5x+ 5 = 0$.

4. Find the sum of roots of a quadratic equation $x^2 + 4x- 32 = 0$

5. Find the product of the roots of the quadratic equation $2x^2 + 7x – 4 = 0$

6. Find the values of K for which the .equation $9x^2 + 2kx + 1 = 0$ have real roots.

7. Find the Value of K if the equation $x^2 - 2(k+1)x + k^2 = 0$ has equal roots.

8. For what value of k, x = a is a solution of equation $x^2 - (a+b)x + k = 0$?

9. Represent the situation in the form of Quadratic equation:

   The Product of Rahman's age (in years) 5 years ago with his age 9 years later is 15.

10. Find the roots of $x^2 - 3x - 10 = 0$

11. The product of two consecutive odd numbers is 483. Find the numbers.

LEVEL -II

1. If the x = 2 and x = 3 are roots of the equation $3x^2 + 2kx - 2m = 0$, find the value of k and m.

2. Solve the equation: $\dfrac{x}{x+1} + \dfrac{x+1}{x} = \dfrac{34}{15}$ x ≠ 0; x ≠ -1.

3. Solve the equation $2x^2 - 5x + 3 = 0$ by the method of completing the square.

4. Using quadratic formula, solve the equation: $p^2x^2 + (p^2 - q^2)x - q^2 = 0$

5. 300 apples are distributed equally among a certain number of student's .Had there been 10 more Students, each would have received one apple less. Find the number of students.

6. Find the roots of Quadratic equation $16x^2 - 24x - 1 = 0$ by using the quadratic formula.

7. Find the Discriminant of the Quadratic equation $2x^2 - 4x + 3 = 0$ and hence find the nature of its roots.

LEVEL – III

1. In a class test ,the sum of Shefali's marks in math's and English is 30.Had she got 2 marks more in math's and 3 marks less in English, the product of their marks would have been 210. Find her marks in the two subjects.

2. A two digit number is such that the product of its digit is 35.When 18 is added to the number, the digits interchange the places. Find the number.

3. Solve $3x^2 - 23x - 110 = 0$

4. Solve the following equation for 'x', $9x^2 - 9(a+b)x + 2a^2 + 5ab + 2b^2 = 0$

5. If the roots of the equation $(a-b)x^2 + (b-c)x + (c-a) = 0$ are equal, prove that 2a = b+c.

Self-Evaluation

1. Find the value of 'p' so that the equation $3x^2 - 5x - 2p = 0$ has equal roots. Also find the roots.

2. The sum of two numbers is 15.If the sum of their reciprocals is $\dfrac{3}{10}$. Find the two numbers.

3. Find the quadratic equation whose roots are $2 + \sqrt{3}$ and $2 - \sqrt{3}$.

4. A person on tour has Rs.360 for his daily expenses. If he exceeds his tour Programme by four days, he must cut down his daily expenses, by Rs 3 per day. Find the number of days of his tour Programme.   33

5. Divide 29 into two parts so that the sum of squares of the parts is 425.

6. Solve for x: $9a^2 - 6ax - (a^2 - b^2) = 0$
7. **If the equation $(1 + m^2)x^2 - 2mcx - c^2 - a^2 = 0$ has equal roots. Show that $c^2 = a^2(1 + m^2)$.**

8. If the price of petrol is increased by Rs.2 per litre, a person had to buy 1 litre less petrol for Rs.1740. Find the Original price of the petrol at that time.
a)   Why do you think the price of petrol is increasing day by day?

b)   What should we do to save petrol?
9. Ramesh wants to design a rectangular park of perimeter 80m and area 400m^2 for jogging and morning walk

for the people of his colony. Is it possible to design the park? If so find the length and breadth of the park.

Which value of Ramesh his depicted here?

Answer
LEVEL-I

1. **2**

2. $\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

3.   **5**
4.   **-4**
5.   **- 2**
6.   **-3$\leq k \leq 3$**
7.   **± 1**
8.   **K=ab**
9. **$x^2 + 4x - 60 = 0$**
10. **-2 and 5**
11. **21,23**

LEVEL-II
1.   **m = 9 and k=-15/2**
2.   **3/2 or -5**
3.   **3/2 or 1**
4.   **q^2/p^2 or -1**
5.   **50**

6. $\dfrac{3 \pm \sqrt{10}}{4}$
7.   **-8, and roots are not real.**

LEVEL-III

1.   (Marks in math's=12, marks in English=18) or (marks in math's=13, marks in English=17)

2.   57

3.   -10/3, 11

4. $\dfrac{a+b}{3}, \dfrac{2(a+b)}{3}$

Self-Evaluation

1. 25/24
2. (10,5)or(5,10)
3. $x^2-4x+1=0$
4. 20 days.
5. (13,16) or (16,13)
6. (a+b)/3 and (a- b)/3
7. Rs 58 per litre
8. Yes,l=20mandb=20m.

ARITHMETIC PROGRESSION

KEY CONCEPT

- An AP is a list of number in which difference of a term to its preceding term is always constant. The constant is called common difference (d) of AP. $d = a_{n+1} - a_n$
- If 'a' is the first term and 'd' is the common difference of an AP, then the AP is a, a+d, a+2d, a+3d…..
- The nth term of an AP is denoted by an
  $a_n = a + (n-1)d$ where a = first term and d = common difference n= number of term
- n^{th} term from the end = l - (n-1)d Where l = last term
- Various terms in an AP can be chosen in following manner.

No. of terms	Terms	Common difference
3	a-d ,a ,a+ d	d
4	a-3d, a-d, a+d, a+3d	2d
5	a-2d, a-d, a, a+d, a+2d	d

- Sum of first n natural number is n (n+1)/2.
- The sum of n terms of an AP with first term a and common difference d is denoted by $s_n = \dfrac{n}{2}\left[2a + (n-1)d\right]$

- The sum of n terms of an AP with first term a and last term l.

  $s_n = \dfrac{n}{2}(a + l)$.

  $a_n = s_n - s_{n-1}$

LEVEL-I

1. Write fourth term of an AP if its nth term is 3n+2.
2. Find A.P which fifth terms are 5 and common difference is –3.
3. Determine the 10th term from the end of the A.P:4,9,14……..254
4. Find whether 0 is a term of the A.P: 40,37,34,31………….
5. Write the value of x for which x+2,2x,2x+3 are three consecutive terms of an A.P
6. Find the sum of first 24 term of AP 5,8,11,14………..
7. Which term of the A.P 12,7,2,-3……is -98
8. The nth term of an A.P is 3n+5 find its common difference.
9. Write the next term of an A.P. $\sqrt{2}$, $\sqrt{18}$
10. If 4/5 ,a,2 three consecutive term of an A.P then find 'a'.

LEVEL- II

11. Find the middle term of A.P 6,13,20,………..216
12. The 6th term of an A.P is -10 and its 10th term is -26.Determine the 15th term of an A.P
13. The 8th term of an A.P is 0 prove that its 38th term is triple its 18th term.
14. The sum of three numbers in A.P is 21and their product is 231 find the numbers.
15. Find the sum of 25th term of an AP which nth term is given by $t_n=(7-3n)$
16. Find the sum of all two digit odd positive numbers
17. Find the sum of three digits numbers which are divisible by 11
18. The sum of first 6 term of A.P is 42.The ratio its 10th term to 30th term is1:3.
    Calculate the first and 13th term of the A.P
19. How many term of the A.P: 17,15,13,…must be added to get the sum 72?
    Explain the double answer.
20. The sums of n,2n,3n term of an A.P are S1,S2,and S3 respectively.Prove that $S_3=3(S_2-S_1)$

LEVEL - III

21. If in an A.P the sum of first m term = n and the sum of 1st n term = m, then Prove that sum of (m+n) term is –(m+n)

22. If $\frac{a^{n+1}+b^{n+1}}{a^n+b^n}$ is the A.M between a and b find the value of n.

23. If the pth , qth, rth term of an A.P be a,b,c respectively then show that a(q-r)+b(r-p)+c(p-q) = 0

24. A man saved Rs 32 during first year Rs. 36 in second year and in this way he increases his saving by Rs. 4 every year find in what time his saving will be Rs 200

25. Find the sum of the following.

$(1-\frac{1}{n})+(1-\frac{2}{n})+(1-\frac{3}{n})+$ ----------- upto nth terms

SELF EVALUATION

26. Find the value of x for A.P, 1+6+11+16...........+x= 148

27. A man repays a loan for Rs 3250 by paying Rs 20 in the first month and then increases the payments Rs15 every month. How long will it take him to clear the loan?

28. If the sum of m terms of an A.P is the same as the sum of its n terms . Show that the sum of its (m+ n) term is zero.

29. Is 51 a term of the A.P, 5,8,11,14,..............

30. . if the mth term of an A.P is 1/n and nth term is 1/m then show that sum of mn term is 1/2(mn+1).

31. If 2x, x+10,3x+2 are in A.P find the value of x.

32. .Find the sum of all 3-digits numbers which are multiple of 7.

33. In an A.P the sum of first n terms is (3n²/2 + 5n/2) .Find its 25th term.

34. The first term of an A.P is -7 and common difference is 5 .Find its 18th term and the general term.

35. Determine the 10th term from the end of the A.P. 4,9,14,.........................254.

36. A sum of Rs 700 is to be used to given 7 cash prizes to the students of a school for their overall academic performance, punctuality, regularity, cleanliness, confidence and creativity and discipline .If each prize is Rs20 less than its preceding prize .Find the value of each of the prizes.

I) which value according to you should be awarded with maximum amount. Justify your answer.

ANSWER

LEVEL-I

1. 14
2. 17
3. 209
4. no
5. 5
6. 948
7. 23rdterm
8. 3
9. $5\sqrt{2}$
10. 7/5

LEVEL- II

11. 111
12. -46
14. (-3,7,11)
13. -800
14. 2475
15. 44550
16. 2, 26
17. 6,12

LEVEL- III

21.

22. **n=0**

23. **..**

24. **5 years**

TRIANGLES

Key Points

Similar Figures: Two figures having similar shapes (size may or may not same), called Similar figures.

Examples :(a)   &   (b)   &   (c)   &

               A pair of Circles   A pair of squares   A pair of Equilateral Triangles
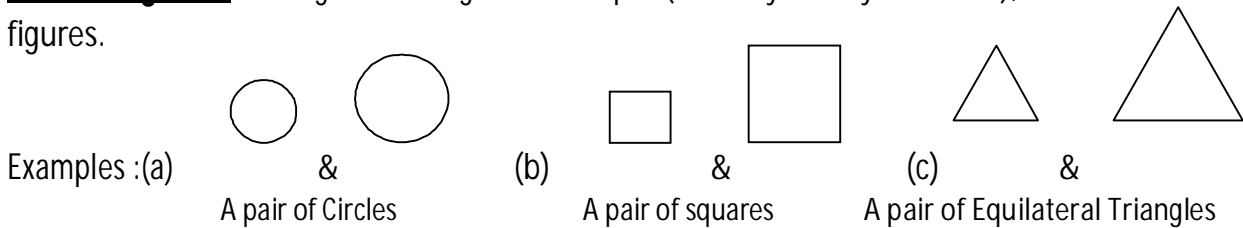
- Pairs of all regular polygons, containing equal number of sides are examples of Similar Figures.
- **SimilarTriangles:** Two Triangles are said to be similar if
  - (a) Their corresponding angles are equal (also called Equiangular Triangles)
  - (b) Ratio of their corresponding sides are equal/proportional
- All congruent figures are similar but similar figures may/may not congruent
- Conditions for similarity of two Triangles
  - (a) AAA criterion/A-A corollary
  - (b) SAS similarity criterion
  - (c) SSS similarity criterion(where 'S' stands for ratio of corresponding sides of two Triangles)

Important Theorems of the topic Triangles

  - (a) Basic Proportionality Theorem(B.P.T.)/Thales Theorem
  - (b) Converse of B.P.T.
  - (c) Ratio of Areas of similar triangles theorem
  - (d) Pythagoras Theorem
  - (e) Converse of Pythagoras Theorem

LEVEL I

(1) In the figure XY ∥QR,PQ/XQ=7/3 and PR=6.3cm then find YR
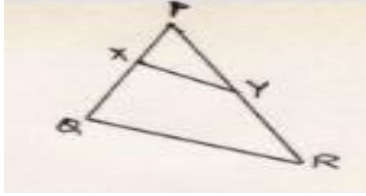
(2) If ΔABC ~ ΔDEF and their areas be 64cm^2 & 121cm^2 respectively ,then find BC if EF=15.4cm

(3) ABC is an isosceles Δ,right angled at C then prove that AB^2=2AC^2

(4) If ΔABC~ΔDEF,∠ A=46^0,∠ E = 62^0 then the measure of ∠ C = 72^0 .Is it true? Give reason.

(5) Theratioofthecorrespondingsidesoftwosimilartrianglesis16:25thenfindtheratioof their perimeters.

(6) A man goes 24km in due east and then He goes 10km in due north. How far is he from the starting point?

(7) The length of the diagonal so far hombus is 16cm & 12cm respectively then find the perimeter of the rhombus.

(8) In the figure LM||CB and LN||CD then prove that AM/AB=AN/AD
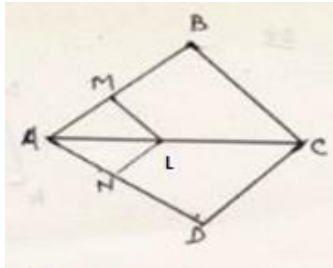
(9) Whichoneisthesidesofarightangledtrianglesamongthefollowing(a)6cm,8cm&11cm (b) 3cm,4cm & 6cm (c) 5cm , 12cm & 13cm

Level II

(1) InthefigureABDisatrianglerightangledatAandACisperpendiculartoBDthenshow that AC^2=BCxDC
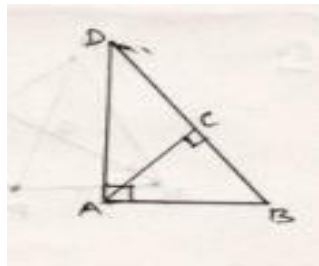
(2) Twopolesofheight10m&15mstandverticallyonaplaneground.Ifthedistancebetween

(3) D &E are the points on the sides AB & AC of ΔABC ,as shown in the figure .If∠B=∠AED then show that ΔABC~ΔAED
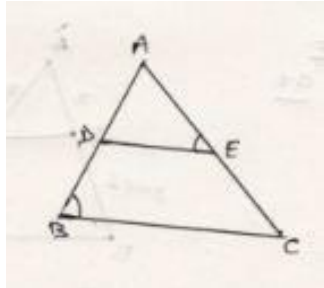
4) In the adjoining figure AB || DC and diagonal AC & BD intersect at point O. If AO=(3x-1)cm,
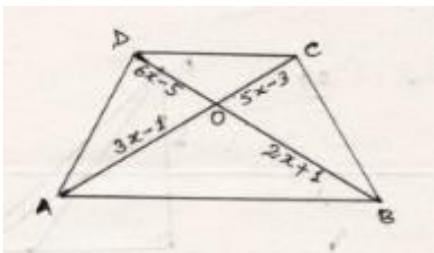
    B=(2x+1)cm ,OC=(5x-3)cm and OD=(6x-5)cm then find the value of x.

5) In the figure D & E trisect BC.Provethat8AE^2=3AC^2+5AD^2

6) In the figure OA/OC=OD/OB then prove
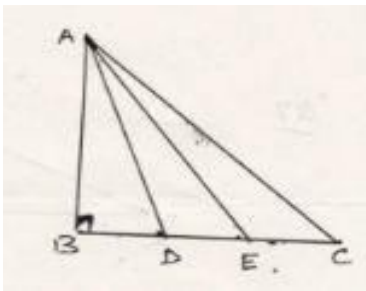
7) that∠A=∠C

8) Using converse of B.P.T. Prove that the line joining the mid points of any two sides of a triangle is parallel to the third side of the triangle.

9) In the given figure ΔABC & ΔDBC are on the same base BC .If AD intersect BC at O then prove that   ar(ΔABC)/ar(ΔDBC)=AO/DO
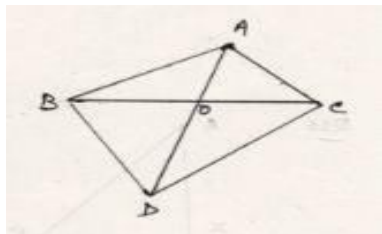
Level III

(1) A point O is in the interior of a rectangle ABCD, is joined with each of the vertices A,B,C&D. Prove that $OA^2 + OC^2 = OB^2 + OD^2$

(2) In an equilateral triangle ABCD is a point on the base BC such that BD=(1/3)BC ,then show that $9AD^2 = 7AB^2$

(3) Prove that in a rhombus, sum of squares of the sides is equal to the sum of the squares of its diagonals

(4) In the adjoining figure ABCD is a parallelogram. Through the mid -point M of the side CD ,a line is drawn which cuts diagonal AC at Land AD produced at E. Prove that EL=2BL
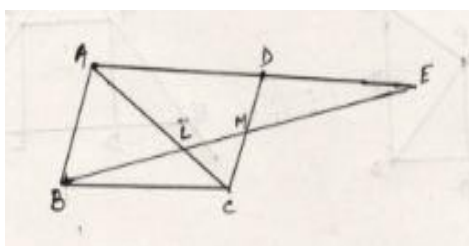
(5) ABC & DBC are two triangle so the same base BC and on the same side of BC with ∠A=∠D=90⁰.If CA &BD meet each other at E then show that AE x EC = BE x ED

(6) ABC is a triangle, right angle at C and p is the length of the perpendicular drawn from C to AB. By expressing the area of the triangle in two ways show that (i) pc = ab (ii) $1/p^2 = 1/a^2 + 1/b^2$
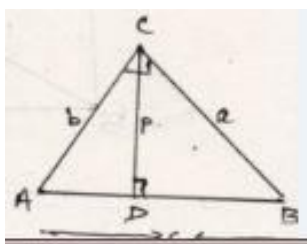
(7) Prove that the ratio of the area so two similar triangles is equal to the ratio of their corresponding sides.

(8)   In the figure AB || DE and BD || EF. Prove that $DC^2 = CF \times AC$
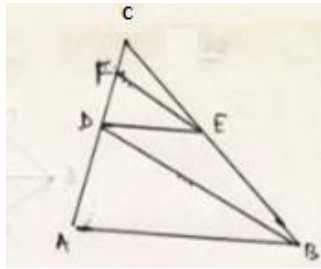
Self-Evaluation Questions including Board Questions

(1) Find the value of x for which DE||BC in the adjoining figure
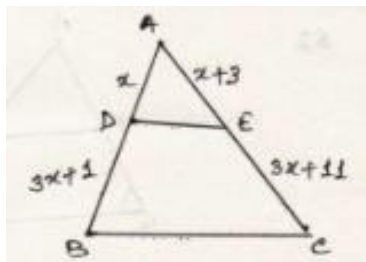
(2) In an equilateral triangle prove that three times the square of one side is equal to four times the square of one of its altitude.

(3) The perpendicular from A on the side BC of a triangle ABC intersect BC at D such that DB=3CD. Prove that $2AB^2 = 2AC^2 + BC^2$

(4) In the adjoining figure P is the mid point of BC and Q is the mid point of AP .If BQ when produced meets AC at R ,then prove that RA=(1/3)CA
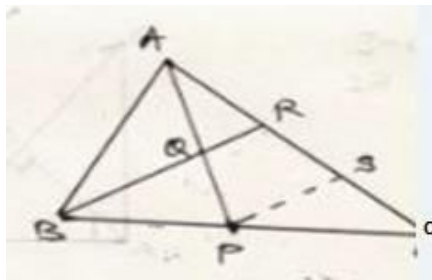
(5) BL and CM are medians of triangle ABC, right angled at A then prove that $4(BL^2 + CM^2) = 5BC^2$

(6) In $\triangle ABC$ if AB = $6\sqrt{3}$cm , AC = 12cm and BC = 6cm then show that $\angle B=90^0$

(7) In the adjoining figure $\angle QRP=90^0, \angle PMR=90^0$, QR = 26cm, PM=8cm and MR=6cm then find the area of $\triangle PQR$
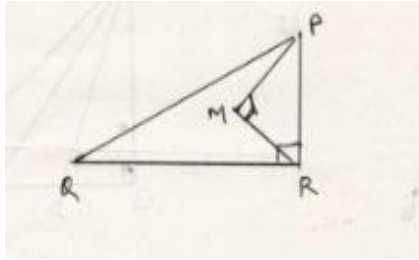

(8) If the ratio of the corresponding sides of two similar triangles is 2:3 then find the ratio of their corresponding altitudes.

(9) In the adjoining figure ABC is a $\triangle$ right angle d at C. P & Q are the points on the sides CA &CB respectively which divides these sides in the ratio 2:1, then prove that $9(AQ^2+BP^2)=13\ AB^2$
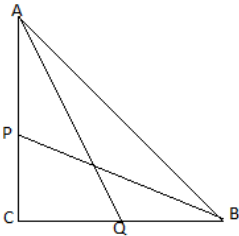
(10) The adjoining figure AB || PQ || CD, AB =x unit, CD =y unit & PQ = z unit then prove that $1/x + 1/y = 1/z$
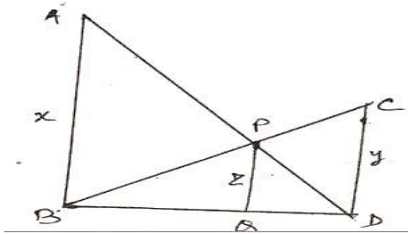
(11) State and prove Pythagoras theorem. Using this theorem find the distance between the tops of two vertical poles of height 12m &18m respectively fixed at a distance of 8m apart from each other.

(12) In the adjoining figure DEFG is a square &$\angle BAC=90^0$ then prove that

(a) $\triangle AGF \sim \triangle DBG$ (b) $\triangle AGF \sim \triangle EFC$ (c) $\triangle DBG \sim \triangle EFC$ (d) $DE^2 = BD \times EC$
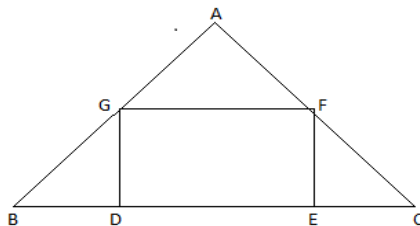
Level I

(1) By B.P.T. PQ/XQ=PR/YR⇨ 7/3=6.3/YR ⇨ YR= 3x6.3/7=2.7

So YR=2.7cm

(2) By theorem   Ar of ΔABC/Ar of ΔDEF= BC²/15.4²

⇨64/121 = BC²/15.4²⇨solving BC = 11.2 cm

(3) By Pythagoras theorem AB²= AC²+BC² ⇨AB²= AC²+AC²(given that AC=BC)

So AB²=2AC²

(4) ΔABC~ΔDEF ⇨ ∠A=∠D=46⁰ , ∠B =∠ E=62⁰ so ∠C =180-(46+62)=72⁰

So it is true.

(5) Let ΔABC~ΔDEF

then AB/DE= BC/EF=AC/DF= perimeter of ΔABC/Perimeter of ΔDEF

⇨AB/DE=perimeter of ΔABC/Perimeter of ΔDEF

So perimeter of ΔABC/Perimeter of ΔDEF=16:25

(6) By Pythagoras theorem , Distance =√ 24²+10²

On Solving , distance =26km

(7) In ΔAOD, by Pythagoras theorem AD=√6²+8²

⇨AD= 10cm

So perimeter of Rhombus = 4x10cm

= 40cm

(8) In ΔABC ,LM//BC so by BPT AM/AB=AL/AC------(i)

Similarly in ΔACD , LN//DC , so by BPT AN/AD = AL/AC-----------(ii)

Comparing results I &ii we get AM/AB= AN/AD

Using Pythagoras thermo ,finding the value of p²+b²&h² separately in each case , it comes equal in case of c where p²+b² comes equal to h²

So sides given in question c is the sides of right triangle

Level II

(1)In ΔABDABC ∠2+∠3=90⁰

⇨∠1+∠2=∠2+∠3

⇨ ∠1=∠3
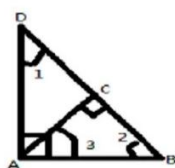
ΔACD~ΔBCA

⇨AC/BC= CD/AC

So AC²=BC x CD

(2)Using Pythagoras theorem

Distance between their tops = $\sqrt{5^2+(5\sqrt{3})^2}$

$\sqrt{25+75}$

Distance between their tops= 10m

(3)In $\triangle AED \& \triangle ABC$

$\angle AED=\angle ABC$(given)

$\angle A=\angle A$(common)

By AA corollary$\triangle ABC \sim \triangle AED$

(4)Diagonals of a trapezium divide each proportionally

So AO/OC =BO/OD

3x-1/5x-3= 2x+1/6x-5

$\Rightarrow 8x^2-20x+8=0$

Solving we get x=2 &1/2(na)

So x=2

(5) BD=DE=EC =P(let)

BE=2P &BC=3P

In Rt $\triangle ABD$,$AD^2= AB^2+BD^2$

$=AB^2+p^2$

In Rt $\triangle ABE$, $AE^2= AB^2+BE^2$

$=AB^2+(2p)^2$

$=AB^2+4p^2$

In Rt $\triangle ABC$,$AC^2=AB^2+BC^2$

$=AB^2+(3p)^2$

$=AB^2+9p^2$

Now taking RHS $3AC^2$+$5AD^2$

$=3(AB2+9p2)+5(AB2+p2)$
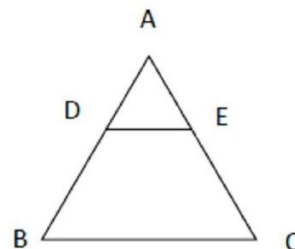
$=8AB^2+32p^2$

$=8(AB^2+4p^2)$

$=8AE^2$

$=LHS$

(6)OA/OC=OD/OB(given)

$\Rightarrow$OA/OD=OC/OB

&$\angle AOD=\angle BOC$(v.o.$\angle^s$)

By SAS similarity condition $\triangle AOD \sim \triangle COB$
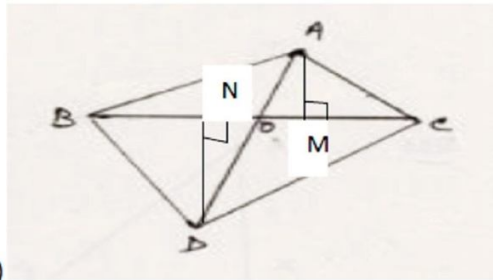
$\Rightarrow \angle A=\angle C$

(7)

Given that AD/DE=1 &AE/EC=1(as D &E are mid points of the sides AB & AC)

$\Rightarrow$AD/DB =AE/EC

By converse of BPT DE//BC

(8)

We draw perpendiculars AM & DN as shown . $\triangle DON \sim \triangle AOM$(by AA corollary)

DN/AM =OD/OA$\Rightarrow$AM/DN=OA/OD-------(i)

Ar of $\triangle ABC$/Ar of $\triangle DBC$ =(1/2xBCx AM)/(1/2x BC x DN)

$\qquad$ =AM/DN

Ar of $\triangle ABC$/Ar of $\triangle DBC$ =AO/OD(from (i))

Level III

(1) We draw PQ ||BC through Pt. O$\Rightarrow$BPQC & APQD are rectangles.

In Rt $\triangle OPB$, by Pythagoras theorem $OB^2=BP^2+OP^2$-------(i)

In Rt $\triangle OQD$,$OD^2=OQ^2+DQ^2$----------(ii)

In Rt $\triangle OQC$,$OC^2=OQ^2+CQ^2$--------(iii)

In Rt $\triangle OAP$, $OA^2=AP^2+OP^2$------------(iv)

On adding (i) &(ii)

$OB^2+OD^2=BP^2+OP^2+OQ^2+PQ^2$

$\qquad =CQ^2+OP^2+OQ^2+AP^2$(BP=CQ & DA =AP)

$\qquad =CQ^2+OQ^2+OP^2+AP^2$

So $\quad OB^2+OD^2$ $=OC^2+OD^2$

(2)
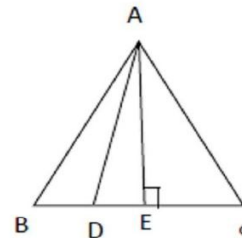
We draw AE perpendicular to BC & AD is joined.

Then BD = BC/3 , DC =2BC/3 & BE=EC =BC/2

In Rt. $\triangle ADE$,$AD^2=AE^2+DE^2$

$\qquad =AE^2+(BE-BD)^2$

$\qquad =AE^2+BE^2+BD^2-2.BE.BD$

$\qquad = AB^2+ (BC/3)^2-2.BC/2.BC/3$

$\qquad =AB^2+BC^2/9-BC^2/3$

$\qquad =(9AB^2+BC^2-3BC^2)/9$

$\qquad -=(9AB^2+AB^2-3AB^2)/9$(Given AB=BC=AC)

$\qquad =7AB^2/9$

$\qquad \Rightarrow 9AD^2=7AB^2$

(3)In Rt.$\triangle AOB$,$AB^2=OA^2+OB^2$

$=(AC/2)^2+(BD/2)^2$

$4AB^2= AC^2+BD^2$---------------(I)

Similarly $4BC^2=AC^2+BD^2$----------------(II)

$4CD^2= AC^2+BD^2$--------------(III)

$4AD^2=AC^2+BD^2$-------------(IV)

Adding these results $4(AB^2+BC^2+CD^2+AD^2) =4(AC^2+BD^2)$

$\Rightarrow(AB^2+BC^2+CD^2+AD^2) =(AC^2+BD^2)$

(4) $\triangle BMC \cong \triangle EDM$ (by ASA criterion)
$\Rightarrow$by cpct DE=BC & AD =BC (opp. sides of //gm)
Adding above results AD+DE=BC+BC
$\Rightarrow$AE =2BC
Now $\triangle AEL \sim \triangle CBL$ (By AA corollary)
EL/BL=AE/BC$\Rightarrow$EL/BL =2BC/BC $\Rightarrow$EL =2BL

(5) $\triangle AEB \sim \triangle DEC$(AA corollary)
AE/DE=EB/EC
$\Rightarrow$AE X EC= BE X ED

(6) Ar of $\triangle ABC$=1/2x AB X DC
$\qquad$ =1/2 X c Xp
$\qquad$ = pc/2
Again Ar of $\triangle ABC$= ½ x AC X BC
$\qquad$ =1/2 x b x a
$\qquad$ = ab/2
Comparing above two areas
ab/2= pc/2
$\Rightarrow$ab=pc
Now in Rt $\triangle ABC$, $AB^2=BC^2+AC_2$
$\qquad$ $c^2=a^2+b^2$
$\qquad$ $(ab/p)^2= a^2+b^2)(ab=pc \Rightarrow c=ab/p)$
$a^2b^2/p^2=a^2+b^2$
$\qquad$ $1/p^2=a^2+b^2/a^2b^2$
$\qquad$ $1/p^2= 1/a^2 +1/b^2$
(7) Theorem question, as proved
(8) In $\triangle ABC$, AB //DE , by BPT AC/DC BC/CE--------(i)
In $\triangle DBC$, EF//BD, by BPT DC/CF = BC/EC -----------(ii)
Comparing (i) &(ii) AC/DC=DC/CF
$\Rightarrow DC^2=AC$ X CF

Self-Evaluation Questions

(1) A/Q AD/DB = AE/EC (by BPT)
$\qquad$ $\Rightarrow$x/3x+1= x+3/3x+11
$\qquad$ $\Rightarrow$3x2+11=3x2+9x+x+3
$\qquad$ So x=3
(2)

In $\triangle ABD$, $AB^2=AD^2+BD^2$
$\qquad$ = $AD^2+(BC/2)^2$(AB=BC=AC)
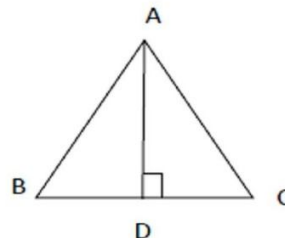
$= AD^2 + AB^2/4$
$4AB^2 = 4AD^2 + AB^2$
$4AB^2 - AB^2 = 4AD^2$
$3AB^2 = 4AD^2$
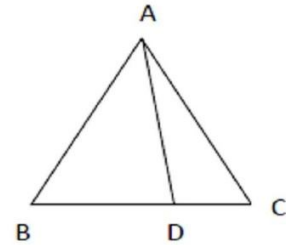
(3), $BC = 4CD \Rightarrow CD = BC/4$
$\Rightarrow BD = 3CD = 3BC/4$ ---------(i)
In $\triangle ABD$, $AB^2 = AD^2 + BD^2$-------(ii)
In $\triangle ACD$, $AC^2 = AD^2 + CD^2$------(iii)
Now $AB^2 - AC^2 = BD^2 = CD^2$
$= 9BC^2/16 - BC^2/16 = BC^2/2$
$2(AB^2 - AC^2) = BC^2$
$2AB^2 = 2AC^2 + BC^2$

(4) we draw PS||BR
In triangle RBC, P is the mid point of BC and PS||BR
RS=CS   [Mid point theorem] ....(1)
In $\triangle$ APS, PS||BR ie PS||QR and Q is the mid point of AP
So AR=RS...........................[II](Mid point theorem)
From results (I)&(II) AR=RS=CS
So AR =1/3AC
(5)
In $\triangle ABL$ $BL^2 = AB^2 + AL^2$
$4BL^2 = 4AB^2 + 4AL^2$
$= 4AB^2 + (2AL)^2$
$4BL^2 = 4AB^2 + AC^2$----(i)
In $\triangle ACM$
$4CM^2 = 4AC^2 + AB^2$----(ii)
On adding
$4BL^2 + 4CM^2 = 4AB^2 + AC^2 + 4AC^2 + AB^2$
$= 5AB^2 + 5AC^2$
$= 5(AC^2 + AB^2)$
$= 5BC^2$
Ie $4BL^2 + 4CM^2 = 5BC^2$
(6)$AC^2 = 12^2 = 144$-----(i)
$AB^2 + BC^2 = (6\sqrt{3})^2 + 6^2$
$= 108 + 36$
$AB^2 + BC^2 = 144$---------(ii)
From (i)&(ii)
$AC^2 = AB^2 + BC^2$(converse of Pythagoras theorem)
$\angle B = 90^0$
(7) In $\triangle PMR$
$PR^2 = PM^2 + MR^2$
$= 6^2 + 8^2$
$= 36 + 64$
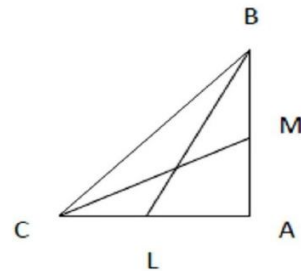
= 100

PR= 10cm

In ΔPQR PQ²= QR²-PR²

$= 26^2 - 10^2$

=676-100

=576

PQ= 24cm

Now Area of ΔPQR= ½ x PR x PQ

= ½ x 10 x24

= 120 cm²

8. Ratio of areas of two similar Δˢ is equal to the ratio of squares of corresponding sides

So Ratio of areas of two similar Δˢ $=(2x/3x)^2 = 4/9$

So Ratio of areas of two similar Δˢ = ratio of squares of their corresponding altitudes= 4/9

So, Ratio of corresponding altitudes = 4/9

9. P divide CA in the ratio 2 :1 Therefore

CP = 2/3 AC ....(i)

QC = 2/3BC ....(ii)

In Right Triangle ACQ,

AQ² = QC² + AC²

Or, AQ² = 4/9 BC² + AC² (QC = 2/3 BC)

Or, 9 AQ² = 4 BC² + 9 AC² ....(iii)

Similarly, In Right Triangle BCP

9BP²= 9BC² + 4 AC² ....(iv)

Adding eq. (iii) & (iv)

9(AQ² + BQ²) = 13(BC²+ AC²)

9(AQ² + BQ²) =13AB²

10.In triangle ABD,

PQ !! AB

PQ/AB= DQ/BD

Or, Z/X=DQ/BD.................................(i)

In triangle BCD,

PQ !! CD

PQ/CD=BQ/BD

Or, Z/Y=BQ/BD................................(ii)

Adding eq. (i) & (ii)

Z/X+ Z/Y = DQ/BD + BQ /BD = DQ + BQ/BD

Or, Z/X + Z/Y = BD/BD=1

Or, 1/X + 1/y = 1/Z

11. State and Prove Pythagoras Theorem

AP = AB – PB = (18- 12)m = 6m

[PB = CD = pm]

Pc = BD = 8m

In ΔACP

AC = √AP² + PC²

= √(8)²+ (6)²

= √64 + 36 = √100 = 10

AC = 10 m

(12)DE//GF &AC cuts them

⇨∠DAG= ∠FGC(corres. ∠^{s})

∠GDE=90^0⇨∠GDA=90^0

ΔADG ∼ΔGCF (By AA corollary ,shown above)

(ii) similarlyΔFEB ∼ΔGCF

Since ΔADG &ΔFEB are both similar to ΔGCF

⇨ΔADG∼ΔFEB

(iii)ΔADG∼ΔFEB

AD/FE=DG/FB

⇨AD/DG= EF/EB

(iv) ΔADG∼ΔFEB

AD/FE=DG/FB

⇨AD/DE= DE/EB(FE=DG=DE)

DE^2=AD X EB

CO-ORDINATE GEOMETRY

BASIC CONCEPTS

1. **Distance Formula:-**

   The distance between two points $A(x_1, y_1)$ and $B(x_2, y_2)$ is given by the formula.

   $$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

   COROLLARY:-The distance of the point $P(x,y)$ from the origin $O(0,0)$ is given by
   $$OP = \sqrt{(X-0)^2 + (Y-0)^2} \quad ie\, OP = \sqrt{X^2 + Y^2}$$

2. **Section Formula :-**

   The co-ordinates of the point $P(x, y)$ which divides the line segment joining $A(x_1, y_1)$ and $B(x_2, y_2)$ internally in the ratio m:n are given by .

   $$x = \frac{mx_2 + nx_1}{m+n} \quad y = \frac{my_2 + ny_1}{m+n}$$

3. **Mid point Formula:-**

   If R is the mid-point, then $m_1 = m_2$ and the coordinates of R are

   $$R\left(\frac{x_1 + x_2}{2} \;,\; \frac{y_1 + y_2}{2}\right)$$

4. **Co-ordinates of the centroid of triangle:-**

   The co-ordinates of the centroid of a triangle whose vertices are $P(x_1, y_1), Q(x_2, y_2)$ and $R(x_3, y_3)$ are

   $$\left(\frac{x_1 + x_2 + x_3}{3} \quad \frac{y_1 + y_2 + y_3}{3}\right)$$

5. **Area of a Triangle:-**
   The are a of the triangle formed a by the points $P(x_1, y_1) Q(x_2, y_2)$ and $R(x_3, y_3)$ is the numerical value of the expression.

   $$ar(\Delta PQR) = 1/2 \left[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) \right]$$

LEVEL - I

1. Find the distance between the points P (7,5) and Q(2,5).
2. If P (α/3 ,4) is the midpoint of the line segment joining the points Q(-6,5) and R(-2,3),then find the value of α.

3. A line intersects y–axis and x-axis at the points P and Q respectively. If (2,-5) is the midpoint of PQ, then find the coordinates of P and Q respectively.

4. If the distance between the points (4,p) &(1,0) is 5, then find the value of p.
5. If the point A(1,2), B(0,0) and C(a ,b) are collinear, then find the relation between a and b.

6. Find the ratio in which the y-axis divides the segment joining (-3,6) and(12,-3).
7. Find the coordinates of a point A, where AB is diameter of a circle whose Centre is(2,-3)and B is(1,4)
8. Find the centroid of triangle whose vertices are (3,-7), (-8,6) and (5,10).

LEVEL - II

1. If A (-2,4),B(0,0),C(4,2) are the vertices of a $\triangle$ABC, then find the length of median through the vertex A.

2. Find the value of x for which the distance between the points P(4,-5) and Q(12,x) is10 units.

3. If the points A (4,3) and B(x,5) are on the circle with Centre O(2,3) then find the value of x.

4. What is the distance between the point A(c,0) and B(0,-c)?

5. For what value of p, are the points (-3,9),(2,p) and(4,-5) collinear?

6. Show that the points (3,2),(0,5),(-3,2) and (0,-1) are the vertices of a square.

7. Point P divides the line segment joining the points A (2,1) and B(5,-8) such that AP:

   AB=1:3 If P lies on the line2x-y+k=0, then find the value of k.

8. Find the relation between x and y if the points(2,1),(x , y)and(7,5) are collinear

LEVEL - III

1. Find the ratio in which the line2x+3y=10 divides the line segment joining the points (1,2) and (2,3).
2. Prove that(4,-1),(6,0),(7,2)&(5,1) are the vertices of a rhombus .Is it a square?
3. Find the area of the triangle formed by joining the mid points of the sides of the triangle whose vertices are (0,-1),(2,1) and (0,3).Find the ratio of this area to the area of the given triangle.
4. Determine the ratio in which the point P(a,-2) divides the line joining of points(-4,3) and B(2,-4).Also find the value of a.

5. If the point C(-1,2) divides internally the line segment joining A(2,5)and in the ratio 3:4. Find the Co- ordinates of B.
6. Show that points (1,1), (4,4) ,(4,8)and (1,5) are the vertices of a parallelogram.
7. Find the value of p, for which the points(-1,3),(2,p) & (5,-1)are collinear
8. If the points(-1,3),(1,-1)and(5,1)are the vertices of a triangle .Find the length of the median through the first vertex.

SELF EVALUATION

1. Find the center of a circle passing through the points (6,-6), (3,-7)and(3,3).
2. If the distance between the points (3,0) and (0,y)is 5 units and y is positive ,what is the value of y?
3. If the points(x ,y),(-5,-2)and(3,-5)are collinear r, then prove that 3x+8y+31=0.
4. Find the ratio in which the Y-axis divides the line segment joining the points (5,-6) and (-1,-4). Also find the coordinates of the point of division.
5. By distance formula, show that the points (1,-1), (5,2) and (9,5) are collinear.
6. Show that the three points (a,a) ,(-a,-a) &(-a√3 , a√3) are the vertices of an equilateral triangle.

Board Questions

Q: 1) Find the value of k, if the point P (2, 4) is equidistant from the points (5, k) and (k, 7).
(CBSE: 2012)
Q:2) If the point A(0,2)is equidistant from the points B (3,p) and C(p,5),find p .Also find the length of AB.
(CBSE: 2014)
Q:3) Find the ratio in which the point P (x,2) divides the line-segments joining the points A(12,5)and B(4,-3).Also ,find the value of x.
(CBSE: 2014)
Q:4) If the points A (-2, 1),B (a, b) and C(4,-1) are collinear and a-b=1.Find the value of a and b.
(CBSE: 2014)
Q: 5) In what ratio does the point (-4, 6) divides the line segment joining the points A (-6, 10) &B (3,-8)
(CBSE: 2012)

ANSWERKEY
LEVEL-I

1. 5
2. -12
3. (0,-10) and(4,0)
4. ±4
5. 2a=b
6. ¼
7. (3,-10)
8. (0,3)

LEVEL-II

1. 5 units
2. 1, -11
3. 2
4. √2 c
5. -1
6. Proof
7. K=-8
8. 4x - 5y - 3=0

LEVEL-III

1. 2:3
2. Proof
3. 1:4
4. a=2/7
5. B(-5,-2)
6. Proof
7. p=1
8. 5

SELFEVALUATION

1. (3,-2)
2. 4
3. Proof
4. 5:1, (0,-13/3)
5. Proof
6. Proof

BOARDQUESTIONS

1.K=3
2. P=1, AB=√10
3. 3:5, x=9
4. a=1, b=0
5.2/7

INTRODUCTION TO TRIGONOMETRY

IMPORTANT CONCEPTS (TAKE A LOOK)

1. TRIGONOMETRY---A branch of mathematics in which we study the relationships between the sides and angles of a triangle, is called trigonometry.

2. TRIGONOMETRIC RATIOS -----Trigonometric ratios of an acute angle in a right triangle express the relationship between the angle and length of its sides.

Trigonometric ratios of an acute angle in a right angled triangle ---

$$\sin \theta = \frac{\text{Side opposite to}\angle\theta}{\text{Hypotenuse}} = \frac{BC}{AC}$$

$$\cos \theta = \frac{\text{Side adjacent to}\angle\theta}{\text{Hypotenuse}} = \frac{AB}{AC}$$

$$\tan \theta = \frac{\text{Side opposite to}\angle\theta}{\text{Side adjacent to }\angle\theta} = \frac{BC}{AB}$$

$$Cot\theta = \frac{1}{\tan\theta} = \frac{\text{side adjacent to }\angle\theta}{\text{Side opposite to}\angle\theta} = \frac{AB}{BC}$$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{\text{Hypotenuse}}{\text{Side adjacent to }\angle\theta} = \frac{AC}{AB}$$

$$\text{cosec}\theta = \frac{1}{\sin\theta} = \frac{\text{Hypotenuse}}{\text{side Opposite to }\angle\theta} = \frac{AC}{BC}$$

For ∠β, sinβ= AB/ AC, cosβ = BC/AC, tanβ= AB/BC
Cosecβ = AC/AB, secβ= AC/BC, cotβ = BC/AB

3. Relationship between different trigonometric ratios

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\cot \theta = \frac{\cos \theta}{\sin \theta}$$

$$\tan \theta = \frac{1}{\cot \theta}$$

$$\cos \theta = \frac{1}{\sec \theta}$$

$$\sin \theta = \frac{1}{\text{cosec}\theta}$$

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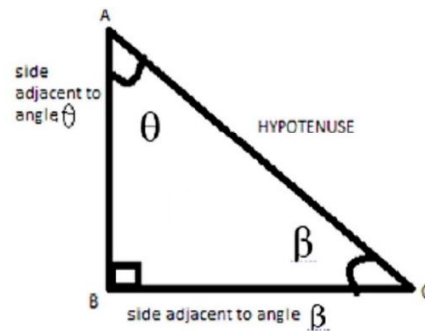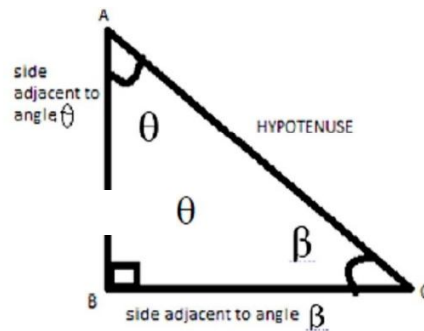
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$$\mathrm{cosec}\theta = \frac{1}{\sin \theta} = \frac{Hypotenuse}{side\ Opposite\ to \angle \theta} = \frac{AC}{BC}$$

For $\angle \beta$, sinβ= AB/ AC, cosβ = BC/AC, tanβ= AB/BC
Cosecβ = AC/AB, secβ= AC/BC, cotβ = BC/AB

3. Relationship between different trigonometric ratios

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\cot \theta = \frac{\cos \theta}{\sin \theta}$$

$$\tan \theta = \frac{1}{\cot \theta}$$

$$\cos \theta = \frac{1}{\sec \theta}$$

$$\sin \theta = \frac{1}{\mathrm{cosec}\,\theta}$$

4. Trigonometric Identity----An equation involving trigonometric ratios of an angle is called a trigonometric identity if it is true for all values of the angle.

  Important trigonometric identities:

  (i) $\sin^2\theta + \cos^2\theta = 1$
  (ii) $1 + \tan^2\theta = \sec^2\theta$
  (iii) $1 + \cot^2\theta = \csc^2\theta$

5. Trigonometric Ratios of some specific angles

Name/angles	0^0	30^0	45^0	60^0	90^0
$\sin\theta$	0	1/2	$1/\sqrt{2}$	$\sqrt{3}/2$	1
$\cos\theta$	1	$\sqrt{3}/2$	$1/\sqrt{2}$	1/2	0
$\tan\theta$	0	$1/\sqrt{3}$	1	$\sqrt{3}$	Not defined
$\cot\theta$	Not defined	$\sqrt{3}$	1	$1/\sqrt{3}$	0
$\sec\theta$	1	$2/\sqrt{3}$	$\sqrt{2}$	2	Not defined
$\csc\theta$	Not defined	2	$\sqrt{2}$	$2/\sqrt{3}$	1

6. Trigonometric ratios of complementary angles.

  (iv) $\sin(90^o-\theta)=\cos\theta$
  (v) $\cos(90^o-\theta)=\sin\theta$
  (vi) $\tan(90^o-\theta)=\cot\theta$
  (vii) $\cot(90^o-\theta)=\tan\theta$
  (viii) $\sec(90^o-\theta)=\csc\theta$
  (ix) $\csc(90^o-\theta)=\sec\theta$

Level – I (These questions can be taken as MCQ/FILL UP/TRUE AND FALSE i.e.1 mark each)

1. If θ and $3\theta-30°$ are acute angles such that $\sin\theta = \cos(3\theta-30°)$, then find the value of $\tan\theta$.   ($1/\sqrt{3}$)

2. Find the value of $\dfrac{\cos\ °+\sin\ °}{1+\cos\ °+\sin\ °}$ ($\sqrt{3}/2$)

3. Find the value of $(\sin\theta+\cos\theta)^2+(\cos\theta-\sin\theta)^2$ (2)

4. If $\tan\theta= \dfrac{3}{4}$ then find the value of $\cos^2\theta-\sin^2\theta$ (7/25)

5. If $\sec\theta + \tan\theta= p$, then find the value of $\sec\theta - \tan\theta$ (1/p)

6. Change $\sec^4\theta-\sec^2\theta$ in terms of $\tan\theta$.   ($\tan^2\theta+\tan^4\theta$)

7. *Prove that* $\dfrac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \tan\theta.$ *(CBSE2009)*

8. In a triangle ABC, it is given that $\angle C=90°$ and $\tan A=1/\sqrt{3}$, find the value of $(\sin A\cos B+\cos A\sin B)$
   (1) (C*BSE2008*)

9. Find the value of $\operatorname{cosec}^2 67°- \tan^2 23°$.   (1)

10. If $\cos x = \cos60°\cos30°+\sin60°\sin30°$, then find the value of x (30^0)

11. If $0° \le x \le 90°$ and $2\sin^2 x = 1/2$, then find the value of x (30^0)

12. Find the value of $\operatorname{cosec}^2 30°-\sin^2 45°-\sec^2 60°$ (1)

13. Simplify $(\sec\theta +\tan\theta)(1-\sin\theta)$ ($\cos\theta$)

14. Prove that $\cos A/(1-\sin A)+\cos A/(1+\sin A)=2\sec A$

Level – II

1. If $\sec\alpha=5/4$ then evaluate $\tan\alpha/(1+\tan^2\alpha)$.

2. If $A+B =90°$, then prove that $\sqrt{\dfrac{\tan A.\tan B+\tan A.\cot B}{\sin A.\sec B} - \dfrac{\sin^2 B}{\cos^2 A}} = \tan A$

3. If $7\sin^2 A +3\cos^2 A= 4$, show that $\tan A =1/\sqrt{3}$.   (CBSE 2008)

4. Prove that $\sqrt{\dfrac{secA-1}{secA+1}} + \sqrt{\dfrac{secA+1}{secA-1}} = 2\ cosec\ A$

5. Prove that $(sin\theta + cosec\theta)^2+(cos\theta+sec\theta)^2=7+tan^2\theta+cot^2\theta$.   (CBSE 2008,2009C)

6. Evaluate: $\dfrac{11sin70^\circ}{7cos20^\circ} - \dfrac{4\ cos53^\circ\ cosec37^\circ}{7\ tan15^\circ\ tan35^\circ\ tan55^\circ\ tan75^\circ}$ Ans: (1)

7. Find the value of $sin30^\circ$ geometrically.   (Not from examination point of view)

8. If $tan\ (A-B) = \sqrt{3}$, and $sin\ (A+B) =1$, then find A and B.   Ans: (A=75, B=15)

9. If θ is an acute angle and $sin\ \theta=cos\theta$, find the value of $3\ tan^2\theta +2\ sin^2\theta -1$.   Ans: 3

10. If $\dfrac{x}{a}\ cos\ \theta+\dfrac{y}{b}\ sin\ \theta = 1$ and $\dfrac{x}{a}\ sin\theta -\dfrac{y}{b}\ cos\ \theta=1$, prove that $x^2/a^2+y^2/b^2=2$.

11. Prove that $\dfrac{sin\theta - 2sin^3\theta}{2cos^3\theta - cos\theta} = tan\ \theta.$

Level - III

1. Evaluate the following:- $sin^2 25^\circ+sin^2 65^\circ+\sqrt{3}\ (tan5^\circ tan15^\circ tan30^\circ tan75^\circ tan85^\circ)$ (2)

2. If $\dfrac{cos\alpha}{cos\beta} = m$ and $\dfrac{cos\alpha}{sin\beta}=n$, show that $(m^2+n^2)\ cos^2\beta=n^2$.   (CBSE2012)

3. Prove that $tan^2\theta+cot^2\theta+2=cosec^2\theta\ sec^2\theta$

4. If $cos\theta+sin\theta=\sqrt{2}cos\theta$, then show that $(cos\theta -sin\theta) =\sqrt{2}sin\theta$.   (CBSE 1997, 2002, 2007)

5. Prove that $(sin\theta+sec\theta)^2+ (cos\theta+cosec\theta)^2=(1+sec\theta cosec\theta)^2$.

6. Prove that $sin\ \theta/ (1-cos\theta) +tan\ \theta/(1+cos\theta)=sec\theta cosec\theta+cot\theta$.

7. If $x=a\ sin\ \theta$ and $y=b\ tan\ \theta$. Prove that $a^2/x^2-b^2/y^2=1$.

8. Prove that $sin^6\theta+ cos^6\theta=1-3sin^2\theta\ cos^2\theta$.

9. Prove that $(sec\ \theta+ tan\ \theta-1)/ (tan\ \theta-sec\theta+1) = cos\ \theta/(1-sin\ \theta)$.

10. Prove that $(1+\cot\theta-\csc\theta)(1+\tan\theta+\sec\theta)=2$ (CBSE2005,07,08)

11. Evaluate: $\dfrac{\sin^2\theta+\sin^2(90^\circ-\theta)}{3(\sec^2 61^\circ-\cot^2 29^\circ)} - \dfrac{3\cot^2 30^\circ\sin^2 54^\circ\sec^2 36^\circ}{2(\csc^2 65^\circ-\tan^2 25^\circ)}$

     Ans: -25/6

12. If $\sin\theta+\cos\theta = m$ and $\sec\theta+\csc\theta=n$, then prove that $n\,(m^2-1)=2m$.

Self-Evaluation

1. If $a\cos\theta+ b\sin\theta = c$, then prove that $a\sin\theta-b\cos\theta= \pm\ \sqrt{a^2+b^2-c^2}$.

2. If A,B,C are interior angle of triangle ABC, show that $\mathbf{cosec^2}\ \left(\dfrac{B+C}{2}\right)$ - $\mathbf{tan^2}\dfrac{A}{2}=1$.

3. If $\sin\theta+\sin^2\theta+\sin^3\theta=1$, prove that $\cos^6\theta-4\cos^4\theta+8\cos^2\theta=4$.

4. If $\tan A=n\tan B$, $\sin A=m\sin B$, prove that $\cos^2 A= (m^2-1)/(n^2-1)$.

5. Prove that $\dfrac{\sec\theta\,\csc(90^\circ-\theta)-\tan\theta\cot(90^\circ-\theta)}{\tan 10^\circ\tan 20^\circ\tan 45^\circ\tan 70^\circ\tan 80^\circ} + \sin^2 55^\circ+ \sin^2 35^\circ =2$

6. If $\sec\theta + \tan\theta=p$, prove that $\sin\theta = (p^2-1)/(p^2+1)$.

7. Prove that $\dfrac{1}{\sec\theta-\tan\theta} - \dfrac{1}{\cos\theta}=\dfrac{1}{\cos\theta} - \dfrac{1}{\sec\theta+\tan\theta}$.
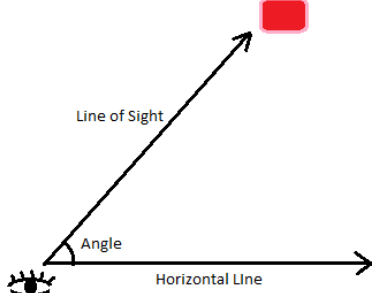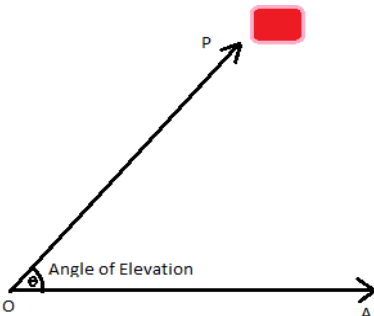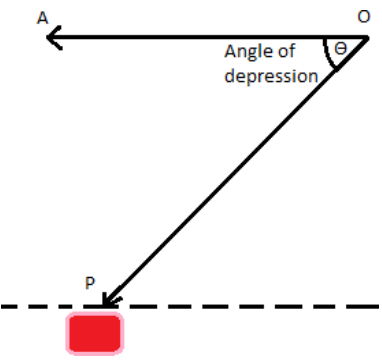
8. Prove that: $\dfrac{\cos\theta}{1-\tan\theta}+\dfrac{\sin^2\theta}{\sin\theta-\cos\theta} = \sin\theta +\cos\theta$

9. Prove that $\dfrac{1+\cos A+\sin A}{1+\cos A - \sin A}=\dfrac{1+\sin A}{\cos A}$.

10. Prove that $(1+\cos\theta+\sin\theta)/(1+\cos\theta -\sin\theta)=(1+\sin\theta)/\cos\theta$

SOMEAPPLICATIONS OF TRIGONOMETRY

HEIGHT AND DISTANCES

KEY POINTS

Line of sight Line segment joining the object to the eye of the observer is called the line of sight.	
Angle of elevation When an observer sees an object situated in upward direction, the angle formed by line of sight with horizontal line is called angle of elevation.	
Angle of depression When an observer sees an object situated in downward direction the angle formed by line of sight with horizontal line is called angle of depression.	

Level – I

1. A pole 3m high casts a shadow $\sqrt{3}$ m long on the ground, then find the sun's elevation?

2. If length of the shadow and height of a tower are in the ratio1:1.Then find the angle of elevation.

3. An observer 1.5m tall is 20. 5 metres away from a tower 22m high. Determine the angle of elevation of the top of the tower from the eye of the observer.

4. A ladder 15m long just reaches the top of vertical wall. If the ladder makes an angle 60^0 with the wall, find the height of the wall.

5. In a rectangle ABCD, AB=20cm，$\angle$BAC=60^0 then find the length of the side AD.

6. Find the angle of elevation of the sun's altitude when the height of the shadow of a vertical pole is equal to its height:

7. From a point 20 m away from the foot of a tower, the angle of elevation of top of the tower is 30°, find the height of the tower.

8. In the adjacent figure, what are the angles of elevation and depression of the top and bottom of a pole from the top of a tower h (m) high:
   Ans 45^0, 60^0
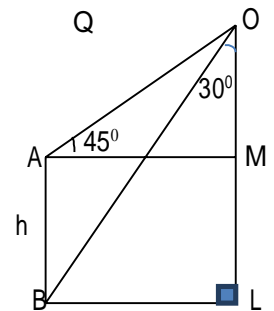
LEVEL –II

9. The length of the shadow of a pillar is $\sqrt{3}$ times its height. Find the angle of elevation of the source of light.

10. A vertical pole10m long casts a shadow$\sqrt{3}$m long. At the same time tower casts a shadow 90m long. Determine the height of the tower.

11. A ladder 50m long just reaches the top of a vertical wall. If the ladder makes an angle of 60^0 with the wall, find the height of the wall.

12. Two poles of height 6m and 11m stands vertically on the ground. If the distance between their feet is12m. Find the distance between their tops.

13. The shadow of tower, when the angle of elevation of the sun is 45° is found to be 10m longer than when it is 60°.Find the height of the tower.

LEVEL –III

14. The angle of depression of the top and bottom of a tower as seen from the top of a100 m high cliff are 30^0 and 60^0 respectively. Find the height of the tower.

15. From a window (9m above ground) of a house in a street, the angles of elevation and depression of the top and foot of another house on the opposite side of the street are 30^0 and 60^0 respectively. Find the height of the opposite house and width of the street.

16. From the top of a hill, the angle of depression of two consecutive kilometer stones due east are found to be 30^0 and 45^0.Find the height of the hill.

17. Two poles of equal heights are standing opposite each other on either side of the road, which is 80m wide. From a point between the road the angles of elevation of the top of the poles are 60^0 and 30^0. Find the heights of pole and the distance of the point from the poles.

18. The angle of elevation of a jetfighter from a point A on the ground is 60^0.After a flight of 15 seconds, the angle of elevation changes to 30^0. If the jet is flying at a speed of 720km/hr, find the constant height at which the jet is flying.

19. A window in a building is at a height of 10 m above the ground. The angle of depression of a point P on the ground from the window is 30^0.The angle of elevation of the top of the building from the point P is 60^0. Find the height of the building.

20. A boy, whose eye level is 1.3 m from the ground, spots a balloon moving with the wind in a horizontal line at same height from the ground. The angle of elevation of the balloon from the eyes of the boy at any instant is 60^0.After 2 seconds, the angle of elevation reduces to 30^0 if the speed of the wind at that moment is $\sqrt{3}$ m/ then find the height of the balloon from the ground.

21. A man on the deck on a ship14m above water level observes that the angle of elevation of the top of a cliff is 60^0 and the angle of depression of the base of the cliff is 30^0.Calculate the distance of the cliff from the ship and the height of the cliff.

22. A tower is 50m high. It's shadow is x m shorter when the sun's altitude is 45° than when it is 30°. Find x correct to the nearest 10.

SELF EVALUATION / HOTS

23. An airplane when flying at a height of 3125m from the ground passes vertically below another Plane at an instant when the angle of elevation of the two planes from the same point on the ground are 30° and 60° respectively. Find the distance between the two planes at that instant.

24. From the top of a building 60m high, the angels of depression of the top and bottom of a vertical lamp post are observed to be 30° and 60°respectively. Find
   [i] horizontal distance between the building and the lamp post
   [ii] height of the lamp post.

25. A vertical tower stands on a horizontal plane and is surmounted by a vertical flagstaff of height 'h' m. At a point on the plane, the angles of elevation of the bottom and the top of the flag staff are

α and β respectively. Prove that the height of the tower is (h tan α) / (tanβ -tanα).

26. The angle of elevation of a cloud from a point 60m above a lake is 30° and the angle of depression of the reflection of the cloud in the lake is 60°. find the height of the cloud from the surface of the lake.

27. A round balloon of radius 'r' subtends an angle α at the eye of the observer whose angle of elevation of centre is β. Prove that the height of the centre of the balloon is (rsinβcosecα/2).

28. A person standing on the bank of a river observes that the angle of elevation of top of building of an organization working for conservation of wildlife. Standing on the opposite bank is 60°.When he moves 40 m away from the bank he finds the angle of elevation to be 30°.Find the height of the building and width of the river.

ANSWERS

Level 1:-

1)60^0 (2) 45^0 (3) 45^0 (4)   7.5 $\sqrt{3}$ m (5) 20 $\sqrt{3}$ m (6) 45^0 (7) 20 /$\sqrt{3}$ m (8) $45^0,60^0$

Level 2:-

(9)30^0 (11) $25\sqrt{3}$ m (12) 13 m (13) $5(\sqrt{3} + 1)$ m
(10)$30\sqrt{3}$ m

Level 3:-

(14) 66.67m (15)   3m ,3 $\sqrt{3}$ m   (16)1.37km (17)   20 $\sqrt{3}$ m, 20m and 16m (18)   500 $\sqrt{3}$ m (19) 30m
(20) 87m   (21)14 $\sqrt{3}$ m and 56m   (22) 36.6m

Self Evaluation

(23) 6250m   (24) 20 $\sqrt{3}$,   40m   (26) 120m   (28) 20 $\sqrt{3}$, 20m

CIRCLES

1. **Circle:** A circle is a collection of all points in a plane which are at a constant distance (radius) from a fixed point (centre).

2. **Secant & Tangent to a Circle:** In fig.1 the line PQ and the circle have no common point. Line PQ is called non-intersecting.Infig.2 line PQ a secant to a circle.Infig.3, there is only 1 point A, which is common to the line PQ and the circle .The line is called a tangent to the circle.
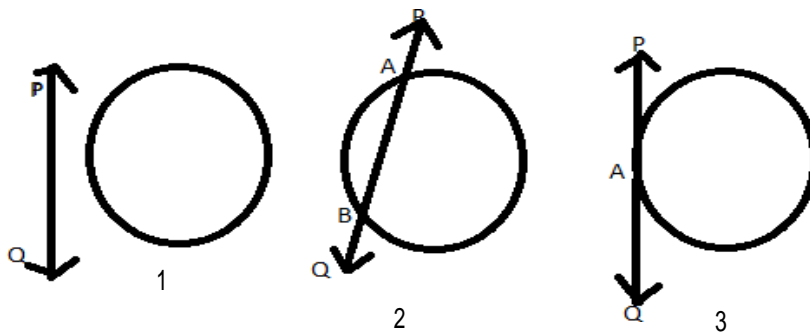
3. **Tangent to a Circle:**
   It is a line that intersects the circle at only one point. There is only one tangent at a point of the circle. The tangent to a circle is a special case of the secant, when the two end points of its corresponding chord coincide.

4. **Theorems:**
   1. The tangent at any point of a circle is perpendicular to the radius through the point of contact.
   2. The lengths of tangents drawn from an external point to a circle are equal.

5. Number of tangents from a point on a circle-

(i) There is no tangent to a circle passing through a point lying inside the circle.

(ii) There is one and only one tangent to a circle passing through a point lying on the circle.

(iii) There are exactly two tangent to a circle through a point lying outside the circle.
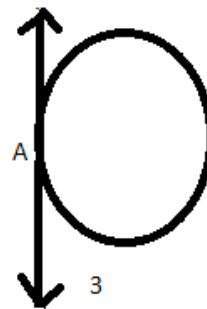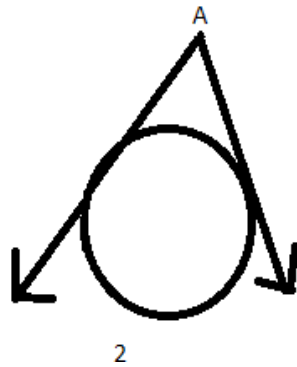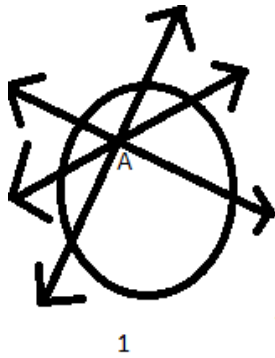
LEVEL I

(These questions can be taken in the form of MCQ/FILL UP/TRUE AND FALSE i.e.1 mark each)

1. In the given fig. O is the centre of the circle and *PQ* is tangent then $\angle POQ + \angle QPO$ is equal to
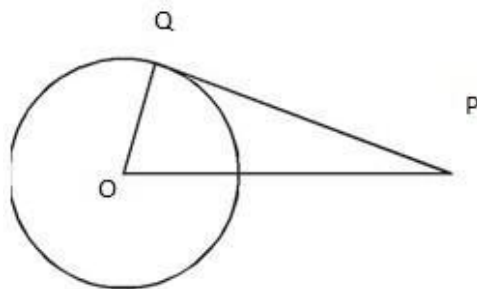
2. If *PQ* is a tangent to a circle of radius 5cm and *PQ* = 12 cm, *Q* is point of contact, then *OP* is

3. In the given fig. *PQ* and *PR* are tangents to the circle, $\angle QOP = 70°$, then $\angle QPR$ is equal to
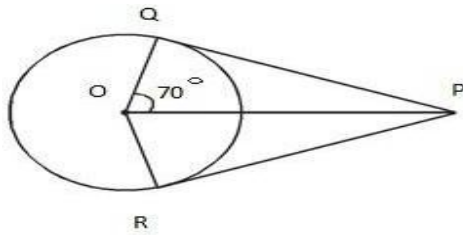
4. In the given fig. *QS* is a tangent to the circle, *OS* = 8 cm, *OQ* = 6 cm then the length of *QS* is
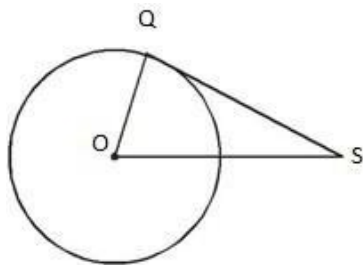
5. In the given fig *PQ* is tangent to outer circle and *PR* is tangent to inner circle. If *PQ* = 4 cm, *OQ* = 3 cm and *OR* = 2 cm then the length of *PR* is
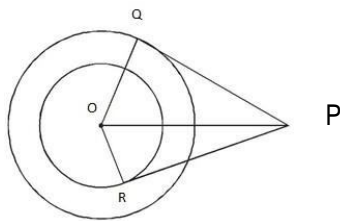
6. In the given fig. *P, Q* and *R* are the points of contact. If *AB* = 4 cm, *BP* = 2 cm then the perimeter of △*ABC* is
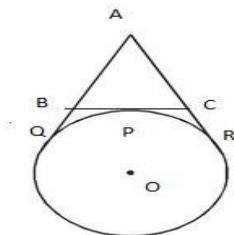
7. The distance between two tangent parallel to each other to a circle is 12cm. The radius of circle is

8. The chord of a circle of radius 10cm subtends a right angle at its centre. Find the length of the chord.
9. How many tangents can a circle have?

10. How many tangents can be drawn from a given exterior point to a circle?

LEVEL - II

11. Two concentric circles of radii a & b (a>b) are given. Find the length of the chord of the larger circle which touches the smaller circle

12. From a point P outside the circle with centre O, tangents PA and PB are drawn to the circle. Prove that OP is the right bisector of the line segment AB.

13. A circles is inscribed in a triangle ABC, touching BC, CA and AB at P, Q and R respectively. If AB= 10 cm AQ =7cm CQ =5 cm. Find BC.
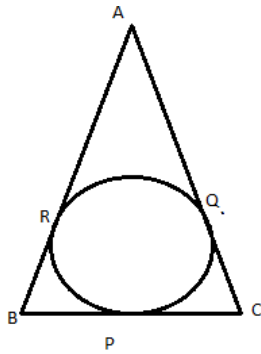
14. A Quadrilateral ABCD is drawn to circumscribe a circle, as shown in the figure. Prove that AB+CD = AD + BC
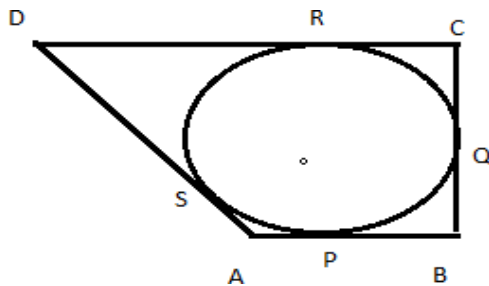
15. Two concentric circles are of radii 7 cm and r cm respectively, where r>7. A chord of the larger circle of length 46 cm, touches the smaller circle. Find the value of 'r'.
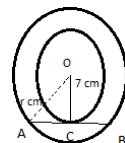
16. Prove that the tangent at any point of a circle is perpendicular to the radius through the point of contact.

LEVEL - III

17. Prove that the length of tangents drawn from an external point to a circle are equal.

18. Prove that the tangents at the extremities of any chord of a circle, make equal angle with the chord.

19. PA and PB are tangents to the circle with the centre O from an external point P, touching the circle at A and B respectively. Show that the quadrilateral AOBP is cyclic.

20. Prove that the parallelogram circumscribing a circle is a rhombus.

21. In the given figure, XY and X'Y' are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersects XY at A and X'Y' at B. Prove that $\angle AOB = 90^0$.
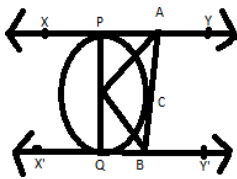
Q.22 Two roads starting from P are touching a circular path at A and B. Sarita runs from P to A, 20km and A to O, 15km and Reeta runs from P to O directly.
(a) Find the distance covered by Reeta.
(b) Who will win the race?
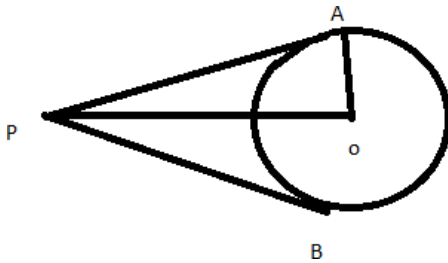
SELF EVALUATION

1. Draw a circle and two lines parallel to a given line such that one is a tangent and the other, a secant to the circle.

2. Prove that perpendicular at the point of contact to the tangent to a circle passes through the centre.

3. Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the lines segment joining the points of contact at the centre.

4. The length of a tangent from a point A at a distance 5cm from the centre of the circle is 4cm.Find the radius of the circle.   Ans:12cm

5. Two concentric circles are of radii 6.5cm and 2.5cm.Find the length of the chord of larger circle which touches the smaller circle.   Ans:3cm

6. From a point P, 10cm away from the centre of the circle, a tangent PT of length 8cm is drawn. Find the radius of the circle.   Ans:6cm

MARKING SCHEME
LEVEL-I

1. 90°

2. $\sqrt{119}$ cm

3. 40°

4. $\sqrt{28}$ cm

5. $\sqrt{21}$ cm

6. 12 cm

7. 6cm

8. $10\sqrt{2}$ cm

9. Infinite

10. Only 2 Tangents

LEVEL-II

11. In Right $\triangle ACO$,
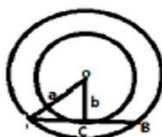
$OA^2 = OC^2 + AC^2$

$AC = \sqrt{a^2 - b^2}$

$AB = 2AC = 2\sqrt{a^2 - b^2}$ [C is midpoint of AB]

12. In $\triangle MAP$ and $\triangle MBP$,

PA=PB [Tangents are equal)

MP=MP(Common)

$\angle MPA \cong \angle MPB$ (By SAS Congruence rule)
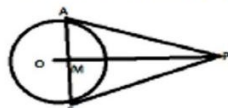
So, MA=MP [CPCT]

And $\angle AMP = \angle BMP$ {CP CT}

BU $\angle AMP + \angle BMP = 180^0$ [Linear Pair]

$\angle AMP = \angle BMP = 90^0$

13. AR=AQ=7cm

BR= (AB-AR) = (10-7) cm = 3cm

BP = BR = 3 cm

CP = CQ=5 cm

BC=BP+CP= (3+5) cm = 8cm

14. AP=AS------------------ (I) [Tangents from A]

BP = BQ------------------ (II)   [Tangents from B]

CR=CQ---------------- (III) [Tangents from C]

DR=DS------------------ (IV) [Tangents from D]

AB+CD= (AP+BP) + (CR+DR)

    (AS+BQ)+ (CQ+DS) (USING I, II, III, IV)

        = (AS+DS) + (BQ+CQ)

        =AS+BC

Hence, AB+CD=AD+BC

15. $\triangle ACO$ we have,

$OA^2 = OC^2 + AC^2$ [By Pythagoras Theorem]

$OA = \sqrt{(OC)^2 + (AC)^2}$

$r = \sqrt{(OC)^2 + (1/2AB)^2}$ [C is mid-point of AB]

$r = \sqrt{7^2 + 23^2}$

$r = \sqrt{578}$

$r = 17\sqrt{2}$ CM

Level III

17. Correct construction
Figure
Proof

18. Correct construction
Figure
Proof

19.
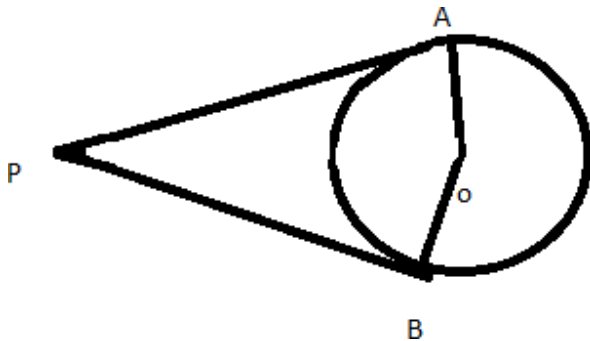
Quad. OAPB,

$\angle AOB + \angle OAP + \angle APB + \angle OBP = 360^0$ Or,

$\angle AOB + 90^0 + \angle APB + 90^0 = 360^0$

Or, $\angle AOB + \angle APB + 180^0 = 360^0$ Or,

$\angle AOB + \angle APB = 180^0$

Hence, quad. OAPB is cyclic.

20.
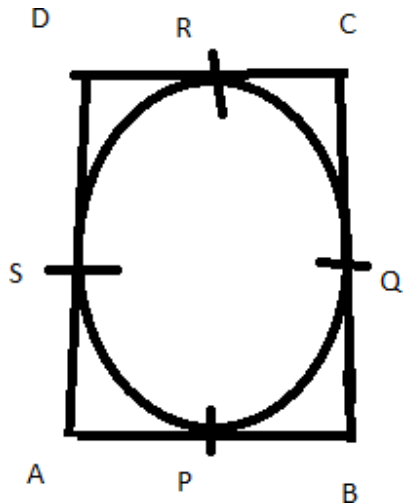
AP=AS ……………(i) [Tangents from A]

BP=BQ…………..(ii)[Tangents from B]

CR=CQ……………(iii)[Tangents from c]

DR=DS……………(iv)[Tangents from D]

Now, AB+CB=AP+BP+CR+DR

$\qquad$ = AS + BQ + CQ + DS [From (i), (ii), (iii), (iv)]

$\qquad$ = (AS + DS) + (BQ + CQ)

$\qquad$ =AD+BC

Or, AB+CD=AD+BC Or,

2AB =2AD

Or, AB = AD

Hence, AB = BC = CD = AD

Hence, ABCD is a rhombus.

21. In quad. APQB

$\angle APO + \angle BQO + \angle QBC + \angle PAC = 360^0$ Or,$90^0 + 90^0 + \angle QBC + \angle PAC = 360^0$

Or,$\angle QBC + \angle PAC = 180^0$...............................(i) We have,$\angle CAO = \frac{1}{2}\angle PAC$

And $\angle CBO = \frac{1}{2} \angle QBC$

Now, $\angle CAO + \angle CBO = \frac{1}{2} (\angle PAC + \angle QBC)$

$\qquad$ = $\frac{1}{2} \times 180^0$ (from eq. i)

$\qquad$ =90^0……………………………… (ii)

In triangle AOB,

$\angle CAO + \angle AOB + \angle CBO = 180^0$

Or, $\angle AOB + 90^0 = 180^0$ (from eq. ii)

Or,∠AOB=90⁰22.(i)
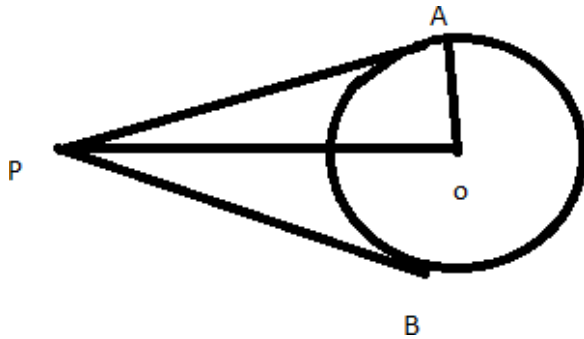
In triangle OAP,
$OP^2=OA^2+AP^2$ (By Pythagoras Theorem)
Or, $OP^2=(15)^2+(20)^2$
Or, $OP^2=625$ Or, OP=25km

(ii) Distance covered by Reeta = 25 km Distance covered by Sarita=20km +15km
= 35 km
So, Rita will win the race.

CONSTRUCTIONS

1. Division of a line segment in the given ratio.

2. Construction of triangles:-

    a. When three sides are given.
    b. When two sides and included angle given.
    c. When two angles and one side given.
    d. Construction of a right angled triangle.

3. Construction of triangle similar to a given triangle as per given scale factor.

4. Construction of tangents to a circle.

EXPECTED LEARNING OUTCOMES

1. Correct use of Mathematical instruments.
2. Drawing a line segment and an angle as per the given data.
3. To divide the given line segment in the given ratio accurately.
4. Neatness and accuracy in drawing.
5. The concept of similar triangles.
6. To Construct a triangle as per the conditions given.
7. To construct similar triangle to a given triangle as per the given ratio.
8. To know that when the ratio is a proper fraction then the similar triangle lies inside the given Triangle and when improper then the similar triangle lies outside the given triangle.
9. To construct tangents to a circle from an external point given.

CONCEPT MAP
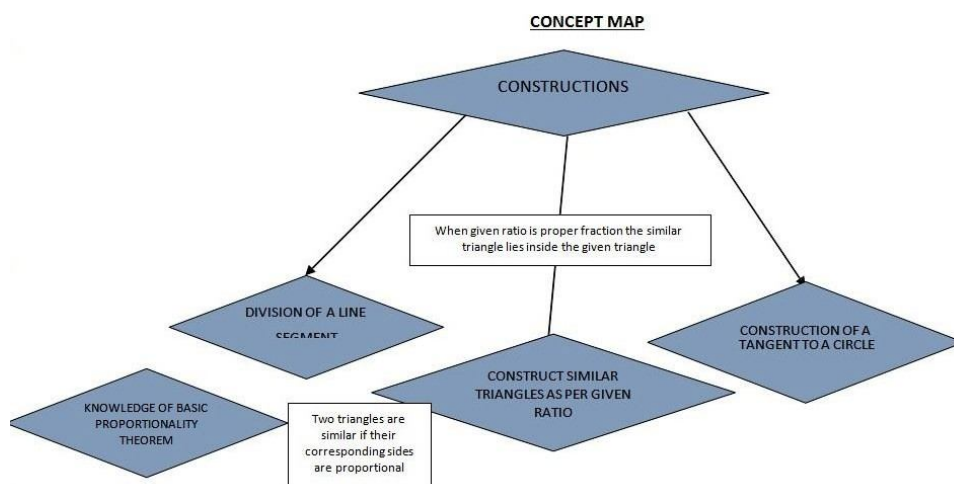
LEVEL – I

1. Draw a line segment AB=8cm and divide it in the ratio 4:3.

2. Divide a line segment of 7cm internally in the ratio2:3.

3. Draw a circle of radius 4cm.Take a point P on it. Draw tangent to the given circle at P.

4. Construct an isosceles triangle whose base is7.5cm and altitude is 4.2cm.

5. Draw a line segment of length 9cm.and divide it in seven equal parts.

LEVEL –II

1. Construct a triangle of sides 4cm, 5cm and 6cm and then construct a triangle similar to it whose sides are 2/3 of the corresponding sides of the first triangle.   (CBSE2013)

2. Construct a triangle similar to a given $\triangle$ABC such that each of its sides is $2/3^{rd}$ of the corresponding sides of $\triangle$ABC. It is given that AB=5cm BC=6cm and AC=7cm. Also write the steps of construction.

3. Draw a pair of tangents to a circle of radius 4cm, which are inclined to each other at an angle of 60^0. (CBSE2013)

4. Draw a circle of radius 5cm. From a point 8cm away from its centre construct the pair of tangents to the circle and measure their lengths.

5. Construct a triangle PQR in which QR=6cm, $\angle$Q=60^0 and $\angle$R=45^0.Construct another triangle similar to $\triangle$PQR such that its sides are 5/6 of the corresponding sides of $\triangle$PQR.

6. Draw a line segment AB=7.5cm and locate a point P on AB such that AP=3/7AB.Give justification of the construction.

LEVEL-III

1. Draw a circle with centre O and radius 3.5cm. Take a horizontal diameter. Extend it to both sides to point P and Q such that OP=OQ=7cm.Draw tangents PA and QB, one above the diameter and the other below the diameter. Is PA||BQ.

2. Construct a ∆ABC in which AB=6cm,⬜A=30°and⬜B=60°.Construct another ∆AB'C 'similar to ∆ABC with base AB'=8cm.   (CBSE2015)

3. Draw a right triangle ABC in which∠B=90^0,AB=5cm,BC=4cm,then construct another triangle A'BC' whose sides are 5/3 times the corresponding sides of ∆ABC.Is the new triangle also a right triangle?

4. Draw a line segment AB of length 8cm.Taking A as centre, draw a circle of radius 4cm and taking B as centre, draw another circle of radius 3cm.Construct tangents to each circle from the centre of the other circle.

5. Draw a line segment AB of length 7cm.Using ruler and compasses, find a point P on AB such that AP/AB= 3/5. (CBSE2011)

6. Construct an isosceles triangle whose base is 8cm.and altitude 4cm. and then construct another triangle Whose sides are ¾ times the corresponding sides of the isosceles triangle. (CBSE 2011)

7. ABC is a right triangle in which AB=5.4cm, BC=7cm and ∠B=90^0.Draw BD perpendicular on AC and a circle through B, C ,D. Construct a pair of tangents from A to this circle.

8. Construct a triangle ABC in which AB=5cm, ∠B=60^0 and altitude CD=3cm.Construct a triangle PQR similar to ∆ABC such that each side of ∆AQR is 1.5 times that of the corresponding sides of ∆ABC.

9. Construct a tangent to a circle of radius 3.5 from a point on the concentric circle of radius6.5cmand measure its length. Also, verify the measurement by actual calculation.

Self-Evaluation

1. Draw a line segment of length 7cm. Find a point P on it which divides it in the ratio 3:5.

2. Draw an isosceles triangle ABC in which AB=AC=6cm and BC=5cm.Construct a triangle PQR similar to $\triangle$ABC in which PQ=8cm. Also justify the construction.

3. Two line segments AB and AC include an angle of 60^0 where AB=5cmand AC=7CM.Locate points P and Q on AB and AC respectively such that AP=3/4AB and AQ=1/4AC.Join P and Q and measure the length PQ.

4. Draw a triangle ABC in which AB=4cm, BC=6cm and AC=9cm.Construct a triangle similar to $\triangle$ABC with scale factor 3/2.Justify your construction.

5. Draw a pair of tangents to a circle of radius 4.5cm, which are inclined to each other at an angle of 45^0.

6. Draw a line segment AB of length 7cm.Taking A as centre, draw a circle of radius 3cm and taking B as centre another circle of radius 2.5cm.Construct tangents to each circle from the centre of the other circle.

7. Two trees are to be planted at two positions A and B in the middle of a park and the third tree is to be planted at a position C in such a way that AC: BC=3:4.How it can be done?

8. Draw a circle of radius 5cm.Draw tangents from the end points of its diameter. What do you observe?
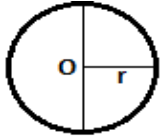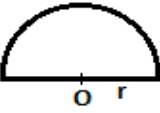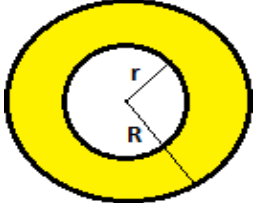
AREAS RELATED TO CIRCLES

KEY POINTS

1. Circle: The set of points which are at a constant distance from a fixed point in a plane is called a circle.
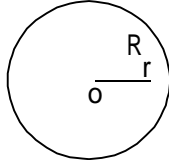
2. Circumference: The perimeter of a circle is called its circumference.
3. Secant: A line which intersects a circle at two points is called secant of the circle.
4. Arc: A continuous piece of circle is called an arc of the circle.
5. Central angle :- An angle subtended by an arc at the center of a circle is called its central angle.

6. Semi-Circle:-A diameter divides a circle in to two equal arcs. Each of these two arcs is called a semi- circle.
7. Segment :- A segment of a circle is the region bounded by an arc and a chord, of a circle.
8. Sector of a circle: The region enclosed by an arc of a circle and its two bounding radii is called a sector of the circle.
9. Quadrant :- One fourth of a circle/circular disc is called a quadrant. The central angle of a quadrant is 90^0.

S.N	NAME	FIGURE	PERIMETER	AREA
1.	Circle		2πr or πd	πr^2
2.	Semi- circle		$\pi r + 2r$	1/2 πr^2
3.	Ring (Shaded region)		2π(r+R)	$\pi(R^2 - r^2)$
4.	Sector of a circle		L+2r=$\frac{\theta}{180}\pi$r+2r	$\frac{\theta}{360}\pi r^2$
5.	Area of Segment of a circle		$\frac{\theta}{180}\pi$r+2r sin$\frac{\theta}{2}$	$\frac{\theta}{360}\pi r^2-\frac{1}{2}r^2\sin\theta$

a. Length of an arc AB$=\frac{\theta}{360}2\pi r$
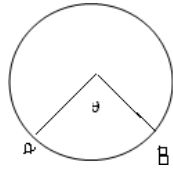
b. Area of major segment=Area of a circle−Area of minor segment

c. Distance moved by a wheel in 1 rotation=circumference of the wheel

d. Number of rotation in 1 minute=Distance moved in 1 minute/circumference

LEVEL-I

1. If the perimeter of a circle is equal to that of square, then the ratio of their areas is

    i.   22/7
    ii.   14/11
    iii.   7/22
    iv.   11/14

2. The area of the square that can be inscribed in a circle of radius 8cm is
    i.   256cm^2
    ii.   128cm^2
    iii.   64√2cm^2
    iv.   64cm^2

3. Area of sector of a circle of radius 36cm is   54 πcm^2.Find the length of the corresponding arc of the circle is

    i. 6π cm

    ii. 3π cm

    iii. 5π cm

    iv. 8π cm

4. A wheel has diameter 84cm.The number of complete revolution it will take to cover 792m is.
   i. 100
   ii. 150
   iii. 200
   iv. 300

5. The length of an arc of a circle with radius 12cm is10 π cm. The central angle of this arc is.
   i. 120^0
   ii. 6^0
   iii. 75^0
   iv. 150^0

6. The area of a circle whose circumference π cm is
   i. 11/2cm^2
   ii. π/4cm^2
   iii. π/2cm^2
   iv. None of these

7. In figure 'o' is the centre of a circle. The area of sector OAB is 5/18 of the area of the circle find x.
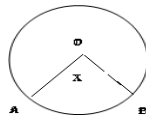
8. If the diameter of a semicircular protractor is 14cm, then find its perimeter.
9. The diameter of a cycle wheel is 21cm.How many revolutions will it make to travel 1.98km?
10. The length of the minute hand of a clock is 14cm.Find the area swept by the minute hand in 5 minutes.

LEVEL – II

1. Find the area of the shaded region in the figure if AC=24cm,BC=10cm and 'o' is the center of the circle (use $\pi = 3.14$)
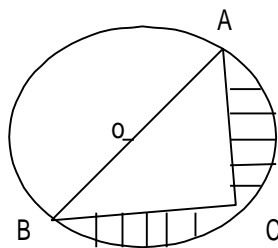
2. The inner circumference of a circular track is 440m.The track is 14m wide. Find the diameter of the outer circle of the track.   [Take π=22/7]

3. Find the area of the shaded region.
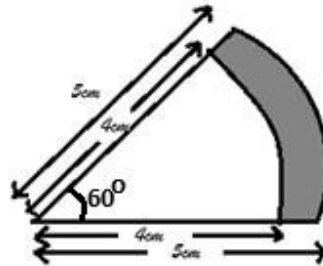
4. A copper wire when bent in the form of a square encloses an area of 121cm². If the same wire is bent in to the form of a circle, find the area of the circle(use π =22/7)

5. A wire is looped in the form of a circle of radius 28cm. It is rebent in to a square form. Determine the side of the square(use π =22/7)

LEVEL-III

1. Three horses are tethered with 7m long ropes at the three corners of a triangular field having sides 20m, 34m 42m. Find the area of the plot.

    i.   Grazed by horses

    ii. Remains un grazed by horses

2.   Calculate the area of shaded region in given figure where ABCD is square of side16cm.
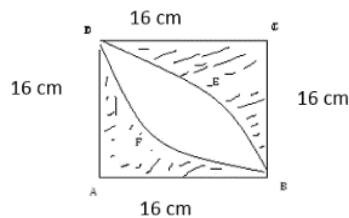
3. ABC is a quadrant of circle of radius14cm and a semi-circle is drawn with BC as diameter. Find the area of Shaded region.
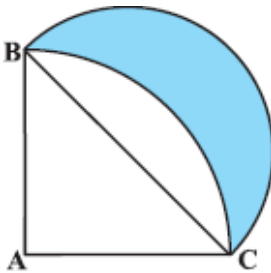
4. The length of a minor arc is 2/9 of the circumference of the circle. Write the measure of the angle subtended by the arc at the centre of the circle.

5. The area of an equilateral triangle is $49\sqrt{3}cm^2$. Taking each angular point as centre, circle is drawn with radius equal to half the length of the side of the triangle. Find the area of triangle not included in the circles. [Take $\sqrt{3}=1.73$]

SELF EVALUATION

1. Two circles touch externally the sum of the areas is $130\pi cm^2$ and distance between there centre is14cm. Find the radius of circle.
2. Two circle touch internally. The sum of their areas is116π cm²distance between their centres is 6cm. Find the radii of circles.
3. A pendulum swings through an angle of 30^0 and describes an arc 8.8cm in length. Find length of pendulum.
4. The side of a square is10cm find the area of circumscribed and inscribed the circle.
5. An Umbrella has 8 ribs which are equally spaced. Assume Umbrella to be flat circle of radius 45cm find the area between two consecutive ribs of umbrella.(use π =3.14)
6. A child prepare a poster on "save energy" on a square sheet whose each side measure 60cm. at each corner of the sheet, she draw a quadrant of radius17.5cm in which she shows the ways to save energy at the centre. She draws a circle of diameter 21cm and writes a slogan in it. Find the area of remaining sheet.(use π=3.14)
7. A birthday cake is circular in shape. This cake is equally divided among six friends where radius of the cake is 60cm.(use π=3.14)
FInd the area of each piece of cake.

ANSWER
LEVEL-I

1. (ii). 14/11
2. (ii). 128 cm^2
3. (ii) 3π cm
4. (iv) 300
5. (iv) 150^0
6. (ii)π/4
7. 100^0
8. 36 cm
9. 3000
10. 154/3 cm^2

LEVEL- II

1. 145.33 cm^2
2. D= 160 m
3. 4.71 cm^2
4. 154 cm^2
5. 44 cm

LEVEL- III

1. (i) 77 m^2
   (ii) 259 m^2
2. 109.7 cm^2
3. 98 cm^2
4. 80^0
5. 7.77 cm^2

SELFEVALUATION

1. 11 cm and 3 cm
2. 4 cm and 10 cm
3. 16.8 cm
4. $50 \pi \text{ cm}^2$, $25 \pi \text{ cm}^2$
5. 794.81 cm^2

6. Area of Remaining sheet=2292.19cm^2
7. Area of each piece=1884cm^2

SURFACE AREA AND VOLUMES

KEY CONCEPTS

1. **CUBOID:**
   - (I) TOTAL SURFACE AREA OF A CUBOID:2(LB+ BH +HL)
   - (II) Volume of a cuboid=L x B x H sq .units
   - (III) Diagonal of cuboid= $\sqrt{l^2} + b^2 + h^2$ units

   CUBE :
   - (IV) Total Surface Area of a Cube = $6a^2$ sq. units
   - (V) Volume of the Cube=a^3 cubic units
   - (VI) Diagonal of cube is= $\sqrt{3}$ a units

2. **Right Circular Cylinder:**
   - (I) Curved Surface Area=$2\pi\pi rh$
   - (II) Total Surface Area=$2\pi r$ (h +r)
   - (III) Volume =$\pi r^2 h$

3. **Right Circular Hollow Cylinder:**
   - (I) Area of each end=$\pi(R^2-r^2)$ [R and r be the external radius and internal radius]
   - (II) Curved Surface Area of Hollow Cylinder=$2\pi h(R+r)$
   - (III) Total Surface Area=$\pi(R + r)$ (2h +R - r)
   - (IV) Volume of material=$\pi h(R^2-r^2)$

4. **Sphere:**
   - (I) Surface Area =$4\pi r^2$
   - (II) Volume= $\frac{4}{3}\pi r^3$

5. **Hemisphere:**
   - (I) Curved Surface Area=$2\pi r^2$
   - (II) Total Surface Area=$3\pi r^2$
   - (III) Volume= $\frac{2}{3}\pi r^3$

6. **Right Circular Cone:**
   - (I) Curved Surface AreaType equation here.=πrl [l=Slant Height]
   - (II) Total Surface Area=πr (l + r)sq units
   - (III) Volume=$\frac{1}{3}\pi r^2 h$

7. **Frustum of a Cone:**
   - (I) Volume of a Frustum of a Cone= $\frac{\pi h (R^2 + r^2 + Rr)}{3}$

   [R – radius of base, r – radius of frustum]   86

(II)   Lateral Surface Area of the Frustum of a cone$=\pi l$ (R+r) [where $L^2=h^2+(R-r)^2$]

(III)   Total Surface Area of the Frustum of the cone$=\pi[R^2+r^2+l$ (R +r)] sq. units

LEVEL WISE QUESTIONS

LEVEL-I

1. The Surface Area of a Sphere is 616cm^2. Find its radius.
2. The slant height of the frustum of a cone is 5cm. if the difference between the radii of its two circular ends is 4cm, write height of the frustum.
3. A cylinder and a cone are of the same base radius and of the same height. Find the ratio of the curved surface area of the cylinder to that of cone.
4. Two cones have their heights in the ratio1:3 and radii 3:1. What is the ratio of their volumes?
5. The radii of two cones are in the ratio 2:1 and their volumes are equal. What is the ratio their heights?
6. The diameter of a sphere is 6cm. it is melted and drawn into a wire of diameter 2mm. Find the length of the wire.
7. Find the curved surface area of a right circular cone of height 15 cm and base diameter is16cm.
8. Find the maximum volume of a cone that can be out of a solid hemisphere of radius r.
9. The diameters of the ends of a frustum of a cone are32cm and 20cm. If its slant height is 10cm. Find the lateral surface area.

LEVEL-II

1. Metallic sphere of radii 6cm, 8cm and10cm respectively, are melted to form a single solid sphere. Find the radius of the resulting sphere.
2. A 20m deep well with diameter7misdugandtheearth from digging evenly spread out to form a platform 22m by14m. Find the height of the platform.
3. Two cubes of volume 64cm^3 are joined end to end. Find the volume of the sphere.
4. The largest sphere is curved out of a cube of a side 7cm. Find the volume of the sphere.
5. A circus tent is cylindrical up to a height of 3m and conical above it. If the diameter of the base is 105 m and the slant height of the conical part is 53m. Find the total canvas used in making the tent.
6. A vessel is in the form of a hemispherical bowl mounted by a hollow cylinder. The diameter of the sphere is 14cm and the total height of the vessel is13cm.Find its capacity?
7. A solid toy is in the form of a right circular cylinder with a hemispherical shape at one end and a cone at the other end. Their common diameter is 4.2cm and the height of the cylindrical and conical position is12cm and 7cm respectively. Find the volume of the solid toy.

8. A solid is in the shape of a cone standing on a hemisphere with both their radii being equal to 1cm and the height of the cone is equal to its radius. Find the volume of the solid in terms of π.

Level-III

1. A hemispherical depression is cut from one face of the cubical wooden block such that the diameter l of the hemisphere is equal to the edge of the cube. Determine the surface area of the remaining solid.
2. A juice seller was serving his customers using glasses. The inner diameter of the cylindrical glass was 5cm, but the bottom of the glass had a hemispherical raised portion which reduced the capacity of the glass. If the height of glass was 10cm, find what the apparent capacity of the glass was and what the actual capacity was.(use $\pi = 3.14$)
3. The height of a cone is 30cm. A small cone is cut off at the top by a plane parallel to the base of its volume be 1/27 of the volume of the given cone, at what height above the base is the section made?
4. An oil funnel of tin sheet consists of a cylindrical portion 10cm long attached to a frustum of a cone. If the total height be 22cm, diameter of the cylindrical portion be 8cm and the diameter of the top of the funnel be18cm. Find the area of the tin required to make the funnel.
5. A solid wooden toy is in the shape of a right circular cone mounted on a hemisphere. If the radius of the hemisphere is 4.2cm and the total height of the toy is 10.2cm. Find the volume of the wooden toy.

SELF-EVALUATION

1. A tent is of the shape of a right circular cylinder up to a height of 3m and then becomes a right circular cone with a maximum height of 13.5m, above the ground. Calculate the cost of painting the inner side of the tent at the rate of Rs.2 per sq. metre, if the radius of the edge is14 metres.
2. A bucket is in the form of frustum of a cone and holds 28.490 litres of water. The radii of the top and bottom are 28cm and 21cm respectively. Find the height of the bucket.
3. The perimeters of the ends of a frustum is48cm and 36cm.If the height of the frustum be 11cm, find its volume.
4. If the radii of the circular ends of a conical bucket which is 45cm high are 28cm and 7cm. Find the capacity of the bucket.

5. A pen stand made of wood is in the shape of a cuboid with four conical depression's to hold pens. The dimensions of the cuboid are 15cm by 10cm by 3.5cm. The diameter of each of the depression is 1cm and the depth is 1.4cm. Find the volume of the wood in the entire stand.
6. Three cubes each of side 5cm are joined end to end. Find the surface area of the resulting cuboid.
7. The diameter of a metallic sphere is 6cm. The sphere is melted and drawn into a wire of uniform cross-section. If the length of the wire is 36m. Find its radius.
8. If the diameter of cross-section of a wire is decreased by 5%. How much percent will the length be increased so that the volume remains the same?

ANSWER
LEVEL-I

1. 7 cm
2. 3cm
3. $\sqrt{2}:1$
4. 3:1
5. 1:4
6. 3600cm or 36m
7. 427.04 cm²
8. $\frac{1}{3}\pi r^3$
9. 816.4cm²

LEVEL- II

1. 12cm
2. 2.5 m
3. 128cm³
4. 179.67cm³
5. 9735m²
6. 1642.67cm³
7. 218.064cm³
8. π

LEVEL- III

1. $\frac{l^2}{4}$ (24+π)
2. 163.54cm³
3. 20 cm
4. 249π
5. 266.12cm³

SELF EVALUATION

1. Total area 1034m²,

   Cost of painting Rs.2068
2. Height 15cm
3. 1554cm³
4. 48510cm³
5. 523.53 cm³
6. 350 cm²
7. 1mm
8. 10.8%

STATISTICS

(i)   Assumed Mean method or Short cut method

Mean=$\bar{X}$=a+$\frac{\sum_{i=1}^{n} f_i d_i}{\sum_{i=1}^{n} f_i}$ Where a = assumed mean And d_i= X_i - a

(ii)   Step deviation method.

Mean= $\bar{x}$=a+$\frac{\sum_{i=1}^{n} f_i u_i}{\sum_{i=1}^{n} f_i} \times h$

Where a = assumed mean h = class size

And u_i= $(X_i – a)/h$

- Median of a grouped frequency distribution can be calculated by Median

=l+$\frac{\left(\frac{n}{2}-c.f\right)}{f} \times h$

Where

l=lower limit of median class n=
number of observations
cf=cumulative frequency of class preceding the median class f=frequency of median class h=class
size of the median class.

- Mode of grouped data can be calculated by the following formula.

Mode=l+ $\left(\frac{f_1-f_0}{2f_1-f_0-f_2}\right) \times h$

Where

l=lower limit of modal class h=size of
class interval
f1 = Frequency of the modal class
fo =frequency of class preceding the modal class
f2=frequency of class succeeding the modal class

- Empirical relationship between the three measures of central tendency.
- 3Median=Mode+2Mean
Or, Mode = 3 Median – 2 Mean

- Ogive
Ogive is the graphical representation of the cumulative frequency distribution. It is of two types:
(i)   Less than type ogive.
(ii)   More than type ogive
   - Median by graphical method
   The x-coordinated of the point of intersection of 'less than ogive' and' more than ogive' gives the
   median.

STATISTICS

(iii) <u>**Assumed Mean method or Short cut method**</u>

Mean=$\bar{X}$= a + $\dfrac{\sum_{i=1}^{n} f_i d_i}{\sum_{i=1}^{n} f_i}$

Where a = assumed mean And $d_i = X_i - a$

(iv) <u>**Step deviation method**</u>.

Mean= $\bar{X}$ =a+ $\dfrac{\sum_{i=1}^{n} f_i u_i}{\sum_{i=1}^{n} f_i}$ x h

Where a = assumed mean h = class size
And $u_i = (X_i - a)/h$

- Median of a grouped frequency distribution can be calculated by Median

$$= \quad l \quad + \quad \dfrac{\frac{n}{2} - CF}{F} \quad x\ h$$

Where
l=lower limit of median class
n=number of observations
cf=cumulative frequency of class preceding the median class
f=frequency of median class
h=class size of the median class.

- Mode of grouped data can be calculated by the following formula.

$$Mode = l + \left(\dfrac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) x\ h$$

Where
l=lower limit of modal class
h=size of class interval
f1 = Frequency of the modal class
fo=frequency of class preceding the modal class
f2=frequency of class succeeding the modal class

- Empirical relationship between the three measures of central tendency.
- 3Median=Mode+2Mean
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  Ogive is the graphical representation of the cumulative frequency distribution. It is of two types:
  (i)   Less than type ogive.
  (ii)   More than type ogive
  - Median by graphical method
    The x-coordinated of the point of intersection of 'less than ogive' and more than ogive' gives the median.

LEVEL – I

S. No	Questions
1	What is the mean of first ten prime numbers?
2	What measure of central tendency is represented by the abscissa of the point where less than ogive and more than ogive intersect?
3	If the mode of a data is 45 and mean is 27, then median is_____.
4	Find the mode of the following

X_i	35	38	40	42	44	
f_i	5	9	10	7	2	

S. No	Questions
5	Write the median class of the following distribution.

Class	0-10	10-20	20-30	30-40	40-50	50-60	60-70
Frequency	4	4	8	10	12	8	4

S. No	Questions
6	The wickets taken by a bowler in 10 cricket matches are as follows:2,6,4,5,0,2,1,3,2,3 Find the mode of the data
7.	How one can find median of a frequency distribution graphically
8.	What important information one can get by the abscissa of the point of intersection of the less than type and the more than type cumulative frequency curve of a group data

LEVEL – II

S.No	Questions	Ans
1	Find the median of the following frequency distribution	167

Height in cm	160-162	163-165	166-168	169-171	172-174
Frequency	15	117	136	118	14

S.No	Questions	Ans
2.	Given below is the distribution of IQ of the 100 students .Find the median of IQ	106.1

IQ	75-84	85-94	95-104	105-114	115-124	125-134	135-144
Frequency	8	11	26	31	18	4	2

S.No	Questions	Ans
3	Find the median of the following distribution	28.5

Class interval	0-10	10-20	20-30	30-40	40-50	50-60
frequency	5	8	20	15	7	5

S.No	Questions	Ans
4	A class teacher has the following absentee records of 40 students of a class for the whole term	

No of days	0-6	6-10	10-14	14-20	20-28	28-38	38-40
No of students	11	10	7	4	4	3	1

Write the above distribution as less than type cumulative frequency distribution

5	Using the assumed mean method find the mean of the following data					27.2
	Class interval	0-10	10-20	20-30	30-40	40-50
	frequency	7	8	12	13	10

6	Name the key terms used in central tendency	Mean Median mode

LEVEL – III

SN	Question	Ans
1	If the mean distribution is 25 <table><tr><td>Class</td><td>0-10</td><td>10-20</td><td>20-30</td><td>30-40</td><td>40-50</td></tr><tr><td>Frequency</td><td>5</td><td>18</td><td>15</td><td>P</td><td>6</td></tr></table> Then find P	P=16
2	Find the mean of the following frequency distribution using step deviation method <table><tr><td>Class</td><td>0-10</td><td>10-20</td><td>20-30</td><td>30-40</td><td>40-50</td></tr><tr><td>Frequency</td><td>7</td><td>12</td><td>13</td><td>10</td><td>8</td></tr></table>	25
3	Find the value of p if the median of the following frequency distribution is 50 <table><tr><td>Class</td><td>20-30</td><td>30-40</td><td>40-50</td><td>50-60</td><td>60-70</td><td>70-80</td><td>80-90</td></tr><tr><td>Frequency</td><td>25</td><td>15</td><td>P</td><td>6</td><td>24</td><td>12</td><td>8</td></tr></table>	P=10
4	Find the median of the following data <table><tr><td>Marks</td><td>Less than 10</td><td>Less than 30</td><td>Less than 50</td><td>Less than70</td><td>Less than 90</td><td>Less than 110</td><td>Less than 130</td><td>Less than 150</td></tr><tr><td>Frequency</td><td>0</td><td>10</td><td>25</td><td>43</td><td>65</td><td>87</td><td>96</td><td>100</td></tr></table>	76.36
5	Compare the modal ages of two groups of students appearing for entrance examination.	A=18.93 B=18.83

Age in yrs	16-18	18-20		20-22	22-24	24-26	
Group A	50	78		46	28	23	

	Group B	54	89	40	25	17		
6	The mean of the following frequency distribution is 57.6 and the sum of the observations is 50. Find the missing frequencies f_1 and f_2.							$f_1 = 8$ and $f_2 = 10$

Class	0-20	20-40	40-60	60-80	80-100	100-120	Total
Frequency	7	f_1	12	f_2	8	5	50

7	The following distribution give the daily income of 65 workers of a factory

Daily income (in Rs)	100-120	120-140	140-160	160-180	180-200
No. of workers	14	16	10	16	9

Convert the above to a more than type cumulative frequency distribution and draw its ogive.

8	Draw a less than type and more than type ogives for the following distribution on the same graph. Also find the median from the graph.

Marks	30-39	40-49	50-59	60-69	70-79	80-89	90-99
No. of students	14	6	10	20	30	8	12

SELF – EVALUATION

1. What is the value of the median of the data using the graph in figure of less than ogive and more than ogive?
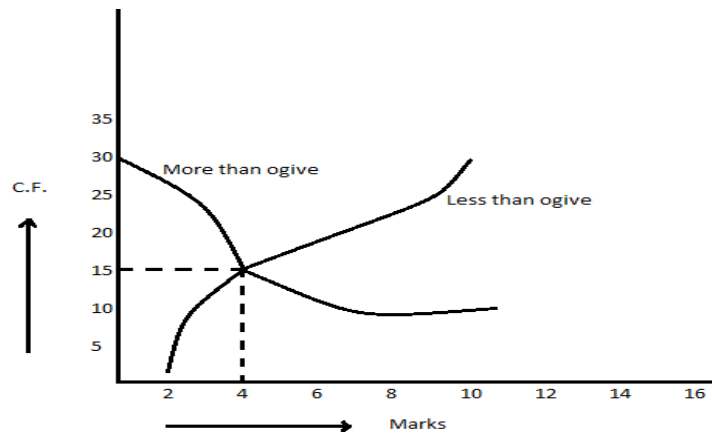
2. If mean=60 and median=50, then find mode using empirical relationship.
3. Find the value of p, if the mean of the following distribution is 18.

Variate (x_i)	13	15	17	19	20+p	23
Frequency (f_i)	8	2	3	4	5p	6

4. Find the mean, mode and median for the following data.

Classes	0-10	10-20	20-30	30-40	40-50	50-60	60-70
frequency	5	8	15	20	14	8	5

5. The median of the following data is 52.5. Find the value of x and y, if the total frequency is100.

Class Interval	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80	80-90	90-100
frequency	2	5	X	12	17	20	Y	9	7	4

6. Draw' less than ogive' and' more than ogive' for the following distribution and hence find its median.

Classes	20-30	30-40	40-50	50-60	60-70	70-80	80-90
frequency	10	8	12	24	6	25	15

7. Find the mean marks for the following data.

Marks	Below 10	Below 20	Below 30	Below 40	Below 50	Below 60	Below 70	Below 80	Below 90	Below 100
No. of students	5	9	17	29	45	60	70	78	83	85

8. The following table shows age distribution of persons in a particular region. Calculate the median age.

Age in years	Below 10	Below 20	Below 30	Below 40	Below 50	Below 60	Below 70	Below 80
No. of persons	200	500	900	1200	1400	1500	1550	1560

9. If the median of the following data is 32.5. Find the value of x and y.

Class Interval	0-10	10-20	20-30	30-40	40-50	50-60	60-70	Total
frequency	x	5	9	12	y	3	2	40

10. The following are ages of 300 patients getting medical treatment in a hospital on a particular day.

Age(in years)	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70
Number of patients	60	42	55	70	53	20

Draw:

1. Less than type cumulative frequency distribution

2. More than type cumulative frequency distribution

Value Based Question

Q1.The following frequency distribution gives the monthly consumption of electricity of 68 consumers of alocality.

Monthly consumption (in units)	65 – 85	85 – 105	105 – 125	125- 145	145- 165	165 – 185	185 – 205
Number of consumers	4	5	13	20	14	8	4

Mr.Sharma always saves electricity by switching off all the electrical equipments just immediately after their uses. So, his family belongs to the group 65-85.

(i)   Find the median of the above data

(ii)   How many families consumed 125 or more units of electricity during a month?

(iii)   What moral values of Mr.Sharma have been depicted in this situation?

Q2.The mileage (km per litre) of 50 cars of the same models is tested by manufacturers and details are tabulated as given below:-

Mileage (km per litre)	10 – 12	12 – 14	14 -16	16- 18
No. of cars	7	12	18	13

ii.   Find the mean mileage.

iii.   The manufacturer claims that the mileage of the model is16km/litre. Do you agree with this claim?

iv.   Which values do you think them manufacturer should imbibe in his life?

ANSWER

1. 12.9

2. MEDIAN

3. 33

4. MODE =40

5. MEDIAN=30-40

6. 2

7. OGIVE

8. Median

Level II

Q1 167

Q2 106.1

Q.3 28.5l

Q4

No. of days	Less Than 6	Less Than 10	Less Than 14	Less Than 20	Less Than 28	Less Than 38	Less Than 40
No. of students	11	21	28	32	36	39	40

Q5 27.2

Q6 Mean, Median, Mode

PROBABILITY

KEY POINTS

1. **Probability:**-The theoretical probability of an event E, written as P(E) is defined as.

   P(E)=<u>Number of out comes Favourable to E</u>

   Number of all possible out comes of the experiment

   When we assume that the out comes of the experiment are equally likely.

2. The probability of a sure event (or certain event) is 1.
3. The probability of an impossible event is 0.
4. The probability of an Event E is number P(E) such that 0≤P(E)≤1.
5. Elementary events:- An event having only one outcome is called an elementary event. The sum of the probabilities of all the elementary events of an experiment is 1.
6. For any event E, P(E)+P($\bar{E}$) = 1, where $\bar{E}$ stands for not E, E and $\bar{E}$ are called complementary event.
7. Performing experiments:-
   a. Tossing a coin.
   b. Throwing a die.
   c. Drawing a card from deck of 52 cards.
8. **Sample space:-**The set of all possible outcomes in an experiment is called sample space.
9. An event is a subset of a sample space.
10. Equally likely events- If one event cannot be expected in preference to other event then they are said to be Equally likely.

LEVEL-I

1. The probability of getting bad egg in a lot of 400 is 0.035. Then find the number of bad eggs in the lot.

2. Write the probability of a sure event.

3. What is the probability of an impossible event?

4. When a dice is thrown, and then find the probability of getting an odd number less than 3.

5. A girl calculates that the probability of her winning the third prize in a lottery is 0.08. If 6000 tickets are sold, how many ticket has she bought.

6. What is probability that a non-leap year selected at random will contain 53 Sundays.

7. A jar contains 54 marbles each of which is blue ,green or white . The probability of selecting a blue marble at random from the jar is 1/3, and the probability of selecting a green marble at random is 4/9 . How many white marbles does the jar contain.

8. two coins are tossed simultaneously. Find the probability of getting exactly one head.

9. A card is drawn from a well shuffled deck of 52cards. Find the probability of getting an ace.

10. In a lottery, there are10 prizes and 25 blanks. Find the probability of getting a prize.

LEVEL-II

1. Find the probability of prime numbers selected at random from the numbers 3,4,5,6...25 .

2. A bag contains 5 red, 4 blue and 3 green balls. A ball is taken out from the bag at random. Find the probability that the selected ball is (a) of red colour (b) not of green colour.

3. A card is drawn at random from a well-shuffled deck of playing cards. Find the probability of drawing

(a) A face card(b)card which is neither a king nor a red card

4. A dice is thrown once. What is the probability of getting a number greater than4?

5. Two dice are thrown at the same time. Find the probability that the sum of two numbers appearing on the top of the dice is more than 9.

6. Two dice are thrown at the same time. Find the probability of getting different numbers on both dice.

7. A coin is tossed two times. Find the probability of getting almost one head.

8. Cards with numbers 2 to 101 are placed in a box. A card selected at random from the box. Find the probability that the card which is selected has a number which is a perfect square.

9. Find the probability of getting the letter M in the word "MATHEMATICS".

LEVEL-III

1. Cards bearing numbers 3,5,7…35 are kept in a bag. A card is drawn at random from the bag. Find the probability of getting a card bearing (a) a prime number less than 15(b) a number divisible by 3 and 5.

2. Two dice are thrown at the same time. Find the probability of getting (a) same no. on the both side(b) different no. on both dices.

3. A child game has 8 triangles of which three are blue and rest are red and ten squares of which six are blue and rest are red. One piece is drawn at random. Find the probability of that is (a) A square (b) A triangle of red colour.

4. Two dice are thrown simultaneously. What is the probability that:

(a) 5 will not come up either of them? (b) 5 will come up on at least one? (c) 5 will come at both dice?

5. The king, queen and jack of clubs are removed from a deck of 52 playing cards and remaining cards are shuffled. A card is drawn from the remaining cards. Find the probability of getting a card of (a) heart(b) queen(c) clubs

6. A game consists of tossing a one-rupee coin 3 times and noting its outcome each time. Hanif wins if all the tosses ,give the same result,i.e.,3 heads or three tails and loses otherwise. Calculate the probability that Hanif will lose the game.

7. Cards bearing numbers 1,3,5…37 are kept in a bag. A card is drawn at random from the bag. Find the Probability of getting a card bearing

(a) A prime number less than15

 (b) A number divisible by 3 and 5.

8. A dice has its six faces marked 0,1,1,1,6,6. Two such dice are thrown together and total score is

recorded.(a) how many different scores are possible? (b) What is the probability of getting a total of seven?

Self-Evaluation/HOTS

1. Two dice are thrown simultaneously. Find the probability of getting an even number as the sum.
2. Cards marked with the number 2 to 101 are placed in a box and mixed thoroughly. One card is drawn from the box. Find the probability that the number on the card is:
      (i)   An even number
      (ii)   A number less than14
      (iii)   A number is perfect square
      (iv)   A prime number less than 20
3. Find the probability that a leap year selected at random will contain 53 Sundays.

Value based Question

Q1.In a survey, it was found that 40% people use petrol, 35% uses diesel and remaining uses CNG for their vehicles.

Find the probability that a person uses CNG at random.

(a) Which fuel out of above 3 is appropriate for the welfare of the society?

Level -I

1. A die is thrown once. What is probability of getting a number greater than4?

2. A bag contains 4 red and 6 blackballs. A ball is taken out of the bag at random. Find the probability of getting a

blackball?

3. A die is thrown once. Find the probability of getting.

   a)   Prime number

   b) A number divisible by 2.

Level -II

1. A bag contains card which are numbered from 2 to 90. A card is drawn at random from the bag. Find

        the probability that it bears.

a.) A Two digit number

b.) A number which is perfect square.

2. Two dice are rolled once. Find the probability of getting such numbers on the two dice whose product is 12.

Level – III

1. Red queens and black jacks are removed from a pack of 52 playing cards. A card is drawn at random from the remaining card, after reshuffling them. Find the probability that the drawn card is:

(i) King   ii) of red colour   iii) a face card iv) queen

2. All the red face cards are removed from a pack of 52 playing cards. A card is drawn at random from the remaining cards after reshuffling them. Find the probability that the card drawn is

(i) Of red colour   ii) a queen   iii) an ace   iv) a face card.

3. In a family of 3 children, find the probability of having at least 1 boy.

4. Three unbiased coins are thrown simultaneously. Find the probability of getting.

i. Exactly two heads.

ii. At least two heads.

iii. At most two heads.

ANSWER

LEVEL-I

1. 14
2. 1
3. 0
4. 1/6
5. 480
6. 1/7
7. 12
8. ½
9. 1/13
10. 2/7

LEVEL - II

1. 8/23
2. A. 5/12 B. ¾
3. A. 3/13 B. 6/13
4. 1/3
5. 1/6
6. 5/6
7. ¾
8. 9/100
9. 2/11

LEVEL– III

1. A.5/17   B.1/17
2. A. 1/6 B. 5/6
3. A. 5/9 B. 5/18
4. A. 25/36 B. 11/36 C. 1/36
5. A. 13/49 B. 3/49, C 10/49
6. ¾
7. A.5/19   B.1/19
8. A. 6 scores B. 1/3

SELF EVALUATION

1. ½
2. A. ½ B 3/25 C 9/100, D. 2/25
3. 2 / 7

VALUE BASED QUESTION

1. Probability =0.25

   CNG

BOARD QUESTION

LEVEL-I

1.1/3
2.3/5
3. ½, ½

LEVEL- II

1. 81/89, 8/89
2. 1/9

LEVEL- III

1. 1/12, 24/48, 1/6, 1/24
2. 10/23, 1/23,2/23,3/23
3. 7/8
4. 3/8, ½, 7/8

★★★★★★★★★★★★★★★★

ACTIVITIES

Activity1:   To find the HCF of two Numbers Experimentally Based on   Euclid Division Lemma

Activity2:   To Draw the Graph of a Quadratic Polynomial and observe:

    i.   The shape of the curve when the coefficient of x^2 is positive

    ii.   The shape of the curve when the coefficient of x^2 is negative

    iii.   Its number of zero

Activity3:   To obtain the zero of a linear Polynomial Geometrically

Activity4:   To obtain the condition for consistency of system of linear Equations in two variables Activity5:

To find geometrically the solution of a Quadratic Equation $ax^2+bx++c=0$, a 0

(where a=1) by using the method of computing the square.

Activity6:   To verify that given sequence is an A .P(Arithmetic Progression) by the paper

Cutting and Paper Folding.

Activity7:   To verify $\sum n = \frac{n(n+1)}{2}$ by Graphical method

Activity8:   To Draw a System of Similar Squares, Using two intersecting Strips with nails

Activity9:   To Draw a System of similar Triangles Using Y shaped Strips with nails

Activity10:   To verify Basic proportionality theorem using parallel line board

Activity11:   To verify the theorem Ratio of the Areas of Two Similar Triangles is Equal to the
Ratio of the Squares of their corresponding sides through paper cutting.

Activity12:   To verify Pythagoras Theorem by paper cutting, paper folding and adjusting (Arranging)

Activity13:   Verify that two figures (objects) having the same shape (and not necessarily the same
size) are similar figures. Extend the similarity criterion to Triangles.

Activity14:   To find the distance between two objects by physically demonstrating the position
of the two objects say two Boys in a Hall, taking a set of reference axes with the
corner of the hall as origin.

Activity15:   Division of line segment by taking suitable points that intersects the axes at some Points
and then verifying section formula.

Activity16:   To verify the formula for the area of a triangle by graphical method.

Activity17:   (a) To make mathematical instrument clinometers (or sextant)for
measuring the angle of elevation/depression of an object

 Activity 18: To calculate the height of an object making use of clinometers (or sextant)

Activity19:   To verify experimentally that the tangent at any point to a circle is
perpendicular to the Radius through that point.

Activity20:   To find the number of tangent from a point to the circle

Activity21:   To verify that lengths of tangents drawn from an external Point to a circle are

                    equal by using method of paper cutting, paper folding and pasting.

Activity22:   To Draw a quadrilateral similar to a given quadrilateral as per given scale factor (Less than1)

Activity23:   To obtain formula for Area of a circle experimentally.

Activity24:   To give a suggestive demonstration of the formula for the surface Area of a circus Tent.

Activity25:   To obtain the formula for the volume of Frustum of a cone.
Activity26:   To find the Average Height (in cm) of students studying in a school.

Activity27:   To Draw a cumulative frequency curve (or an ogive) of less than type.

Activity28:   To Draw accumulative frequency curve (or an ogive) of more than type.

Activity29:   To get familiar with the idea of probability of an event through a double color card experiment.

Activity30:   To verify experimentally that the probability of getting two tails when two coins are tossed

                    simultaneously is ¼ = (0.25) By eighty tosses of two coins.

ORAL TEST (REAL NUMBERS)

Answer the following questions:
1. Euclid's division algorithm is a technique to compute the_____of two given positive integers.
2. HCF (124, 24) is_____.
3. "Every composite number can be expressed (factorised) as a product of primes, and this factorization is unique, apart from the order in which the prime factors occurs". The above statement is called_____.
4. For any two positive integers a and b,
   a x b=HCF(a, b) x_____
5. If a number cannot be written in the form p/q, where p and q are integers and q ≠0, then it is called_____.

QUIZ
(POLYNOMIALS)

Answer the following questions:
1. What is a quadratic polynomial?
2. What is the degree of a quadratic polynomial?
3. What are the zeroes of a polynomial?
4. What is the shape of curve of a quadratic polynomial graph?
5. State remainder theorem.

ORAL TEST

1. If P(x) is a polynomial in x, the highest power of x in P(x) is called the_____of the polynomial P(x).
2. A polynomial of degree 2 is called a_____.
3. The linear polynomial ax + b, a≠0, has exactly one zero, namely, the x-coordinate of the point where the graph of y= ax+ b intersects the_____.
4. A polynomial P(x) of degree n has atmost_____zeroes.
5. The sum and the product of the zeroes of a quadratic polynomial $x^2+7x+10$ is_____ and_____.

QUIZ
(Pair of linear equations in two variables)

Answer the following questions:
1. What is a pair of line of equations in two variables?
2. Give the general form of a pair of linear equation?
3. What are the methods of solving a pair of linear equation in two variables?
4. What is the condition for inconsistent solution?
5. What is the shape of curve in graph of a linear equation?

Oral Test

1. Every solution(x, y) of a linear equation in two variables, ax + by + c = 0 corresponds to a _____ on the line representing the equation, and vice versa.
2. If the pair of linear equations in two variables have only one common point on both the lines, then we have a_____solution.
3. A pair of equations which has no solution is called a/an_____pair of linear equations.
4. Half the perimeter of a rectangular garden, whose length is 4m more than its width is 36m. The dimension of the garden are_____and_____.
5. A pair of linear equations in two variables can be represented and solved by the graphical method and_____method.

QUIZ
(Triangles)

1. What is SAS similarity criterion?
2. What is the relationship between congruency and similarity of figures?
3. What are the criteria for the similarity of two triangles?
4. For what types of triangles is Pythagoras theorem applicable?
5. What is the name of Basic Proportionality Theorem?

ORAL TEST

1. All_____triangles are similar (equilateral / isosceles / Scalene)
2. The longest side of a right angled triangle is called_____.
3. In a _____square of the hypotenuse is equal to the sum of squares of the other two sides.
4. In the given figure, if DE || BC, then the value of x is_____
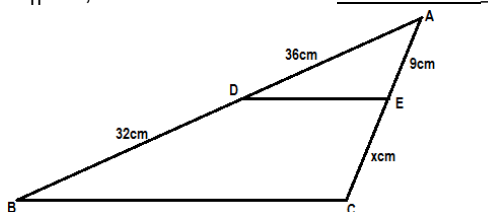
5. State whether the following quadrilateral are similar or not.
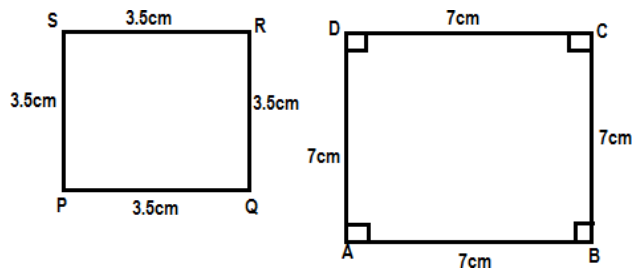
QUIZ
(Introduction to Trigonometry)

1. What is trigonometry?
2. What are trigonometric ratios of an acute angle in a right triangle?
3. From the figure find the value of cos A.
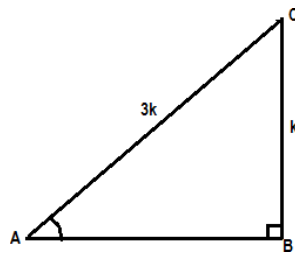
4. Write the trigonometric ratios of 60º.
5. Evaluate tan70°/ cot20°.

ORAL TEST

1. In a right triangle ABC, right angles at B , sin A =_____.
2. Sec (90⁰– A) =_____
3. Sec²A-_____= 1, for $0° \leq A < 90°$.
4. If cot A= 7/8, then (1+ sin A) (1 – sin A)/(1 + cos A)(1 – cos A)
5. (1–tan²45º) / (1+tan²45º) = _____

QUIZ
(STATISTICS)

1. Name the measures of central tendency.

2. What is cumulative frequency?

3. How will you represent the cumulative frequency distribution graphically?

4. How will you find the median of a grouped data graphically with the help of one ogive?

5. How will you find the median of a grouped data graphically with the help of both ogives (i.e of the less than type and of more than type)?

ORAL TEST

1. _____is the sum of the values of all the observations divided by the total number of observations.

2. Class mark = _____/2.

3. The formula for finding the mean using the step deviation method is_____.

4. The formula for finding the mode in a grouped frequency distribution is_____.

5. The formula for finding the median of grouped data is_____.

KENDRIYA VIDYALAYA SANGATHAN
MODEL QUESTION PAPER 1 **(STANDARD)**
2019-20
Blue Print

Class- X   Time-3 Hours
Subject-Mathematics   Marks: 80

SLNO	NAME OF THE CHAPTER	VSA	SA I	SA II	LA	Total
	NUMBER SYSTEM					**6**
1	REAL NUMBERS	1(1)	2(1)	3(1)		6(3)
	ALGEBRA					**20**
2	POLYNOMIALS	1(1)				1(1)
3	PAIR OF LINEAR EQUATIONS IN TWO VARIABLES	1(1)	4(2)	3(1)		8(4)
4	QUADRATIC EQUATIONS	1(1)			4(1)	5(2)
5	ARITHMETIC PROGRESSION	1(1)	2(1)	3(1)		6(3)
	GEOMETRY					**15**
6	TRIANGLES	2(2)			4(1)	6(3)
7	CIRCLES	2(2)		3(1)		5(3)
8	CONSTRUCTIONS				4(1)	4(1)
	TRIGNOMETRY					**12**
9	INTRODUCTION TO TRIGNOMETRY	4(4)		3(1)		7(5)
10	APPLICATIONS OF TRIGNOMETRY	1(1)			4(1)	5(2)
11 12	**STATISTICS** **PROBABILITY**	1(1) 1(1)	2(1)	3(1)	4(1)	11(5)
13	**CO-ORDINATE GEOMETRY**	1(1)	2(1)	3(1)		6(3)
	MENSURATION					**10**
13	SURFACE AREAS AND VOLUMES	2(2)			4(1)	6(3)
14	AREA RELATED TO CIRCLE	1(1)		3(1)		4(2)
	Total	20(20)	12(6)	24(8)	24(6)	80(40)

Note:-Marks are outside the bracket and no. of questions are inside the bracket

Design of Question Paper

Type of question	Marks per question	Total Number of question	Total Marks
VSA	1	20	20
Short Answer 1	2	6	12
Short Answer 2	3	8	24
Long Answer	4	6	24
Total		**40**	**80**

Kendriya Vidyalaya Sangathan
Question Paper Design

S. no	Typology of Question	VSA	SA1	SA2	LA	TOTAL	WEIGHTAGE
1	REMEMBERING	6	2	2	1	20	25%
2	UNDERSTANDING	6	1	1	3	23	29%
3	APPLYING	5	2	2	1	19	24%
4	ANALYZING, EVALUATING & CREATING	3	1	3	1	18	22%
	TOTAL	20x1=20	6x2=12	8x3=24	6x4=24	80	100%

KENDRIYA VIDYALAYA

SANGATHAN MODEL

QUESTION PAPER - 1

2019-20

Mathematics- STANDARD (041)

Class:-X   Time: 3 Hrs

Marks: 80

General Instructions:
  (i)   All questions are compulsory.
  (ii)   The question paper comprises of five sections, A, B, C, D and E. You are to attempt all the sections.
  (iii)   Section A comprises of 10 multiple choice questions (MCQs) one mark each. Section B comprises of 10 questions of one mark each. Section C comprises of 6 questions of 2 marks each. Section D comprises of 8 questions of 3 marks each. Section E comprises of 6 questions of 4 marks each.
  (iv)   There is no overall choice. However, an internal choice has been provided in Section C, D and E.
  (v)   Use of calculator and other electronic device is not permitted.

SECTION – A
Question Numbers 1 to 10 carry one mark each.

1. The product of three consecutive integers is divisible by
    (a) 2   (b) 3   (c) 5   (d) 6
2. The midpoint of the line segment joining the points (-2, 8)and (-6,-4)
    (a) (-4, -6)   (b) (2, 6)   (c) (-4, 2)   (d) (4, 2)
3. If the lines given by 3x+2ky=2 and 2x+5y+1=0 are parallel, then value of k is
    (a)$\frac{-5}{4}$ (b)$\frac{2}{5}$ (c)$\frac{15}{4}$ (d)$\frac{3}{2}$
4. Nature of roots of the equation $x^2 + 2x + 4 = 0$ is
    (a) Real and equal   (b) real and unequal   (c) not real   (d) none of these
5. If n^{th} term of an AP is an + b then its common difference is
    (a) b   (b) a   (c) n   (d) – a
6. Complement angle of Cosine is
    (a)Secant   (b) Sine   (c) Tangent   (d) Cosecant
7. At point A on a diameter AB of a circle of radius 10cm, tangent XAY is drawn in the circle. The length of the chord CD parallel to XY at a distance 16cm from A is
    (a) 8 cm   (b) 10cm   (c) 16cm   (d) 18cm
8. If two towers of height h_1 and h_2 subtend angles 60^0 and 30^0 respectively at the mid -point of line joining their bases then $h_1:h_2$ is
    (a) 3 :1   (b) 1 :2   (c)$\sqrt{3}$: 1   (d) 1 : $\sqrt{3}$
9. A wire can be bent in the form of a circle of radius 7cm. if it bent in the form of a square, then its area will be
    (a) 351cm^2 (b) 516 cm^2 (c) 218 cm^2 (d) 121 cm^2

10. Class mark of the Class 10 -25 is
   (a) 10   (b) 25   (c) 15   (d) 17.5

SECTION – B
Question Numbers 11 to 20 carry one mark each.

The following questions (11 to 15) consist of two statements – Assertion (A) and Reason(R).
Answer these questions selecting the appropriate option given below:
(a)Both A and R are true and R is the correct explanation for A.
(b) Both A and R are true and R is not the correct explanation for A.
   (c)A is true but R is False.
   (d) A is False but R is true.
11. **Assertion(A)**: The line segment joining the mid points of the sides of a triangle form four triangles similar to original triangle.
Reason (R): Two polygons are similar if their corresponding sides are proportional.
12. **Assertion(A)**: If $2 \, Sin2\theta = \sqrt{3}$ then $\theta = 30^0$.
Reason (R): The value of $sin60^0 = \frac{\sqrt{3}}{2}$.
13. **Assertion(A)**: If the surface area of a sphere is 616 cm². Then its radius is 6cm.
Reason (R): Surface area of sphere $= 4\pi r^2 sq \, units$.
14. **Assertion(A)**: A circle can have more than two parallel tangents, parallel to the given chord.
Reason (R): The point at which a line touches the circle, is called the point of contact.
15. **Assertion(A)**: If a number x is chosen at random from the numbers -3,-2,-1,0,1,2,3 then the probability of $|x| < 2 \, is \, \frac{2}{7}$.
Reason (R):$|x|<2 \Rightarrow -2 < x < 2 \Rightarrow x = \{-1,0,1\}, where \, x \, is \, an \, integer$.
Fill in the Blanks. (Q. No. 16 to 20)
16. Acute angle satisfying $\sqrt{3}Sin\theta = Cos\theta$ is ………………..
17. Degree of polynomial $x^3 + 5x^2 - 12x - 2$ is ……………..
18. Volume of a frustum of a cone having base radii r_1 and r_2 and height h is …………..
19. The ratio of the areas of two similar triangles is equal to the square of the ratio of their ………………..
20. The value of $tan^2\theta - \frac{1}{cot^2\theta}$ is …………….

SECTION – C
(Question numbers 21 to 26 carry 2 marks each)

21. Use Euclid's division lemma to show that the square of any positive integer is either of the form 3m or 3m+1 for some integer m.
22. For what value of k,will the system of linear equations $x + 2y = 5$ and $3x + ky - 15 = 0$ has unique solution?
23. Find the value of k for which the lines (k+1) x+3ky+15=0 and 5x+ ky + 5 =0 are coincident.
24. Find how many integers between 200 and 500 are divisible by 8.
OR
The general term of a sequence is given by a_n= 4n + 15. Is the sequence forming an AP? If so, find its 15th term.
25. Find the ratio in which the line segment joining the points (-2, 3) and (3, - 2) is divided by y-axis.
OR
Find the value of y for which the distance between the points P (2, -3) and Q (10, y) is 10 units.
26. A coin is tossed twice. What is the probability of getting at most one tail?   113

SECTION – D
(Question numbers 27 to 34 carry 3 marks each)

27. Prove that $2 - \sqrt{3}$ is an irrational number.

28. Solve the pair of equations: $\dfrac{5}{x-1} + \dfrac{1}{y-2} = 2$; $\dfrac{6}{x-1} - \dfrac{3}{y-2} = 1$

OR

If x +1 is a factor of $2x^3 + ax^2 + 2bx + 1$, then find the values of a and b given that 2a – 3b = 4

29. If A, B and C are the interior angles of ΔABC, then prove that

$$\tan\frac{A+B}{2} = \cot\frac{C}{2}$$

OR

Prove that

$$\frac{sinA}{1+cosA} + \frac{1+cosA}{sinA} = 2\,cosecA$$

30. How many terms of the A.P. 24, 21, 18,…must be taken so that their sum is 78?.

31. If the points (-2,-1), (1, 0), (x, 3) and (3, y) form a parallelogram, find the values of x and y.

OR

The co-ordinates of the points A, B and C are (6, 3), (-3, 5) and (4,-2) respectively. P(x, y) is any point in the plane. Show that $\dfrac{ar(\Delta PBC)}{ar(\Delta ABC)} = \dfrac{x+y-2}{7}$.

32. Prove that the tangents drawn from an external point to the circle are equal in length.

33. Find the area of the shaded region in the given figure. If O is the centre of the two concentric circles and radii of the circles are 7cm and 14 cm respectively along with angle AOC =40⁰.
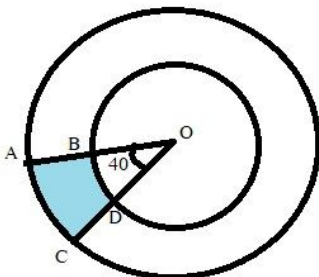
34. Cards numbered from 11 to 60 are kept in a box. If a card is drawn at random from the box, find the probability that the number on the drawn card is:

(i) an odd number   (ii) a perfect square number   (iii) a prime number less than 20.

SECTION – E
(Question numbers 35 to 40 carry 4 marks each)

35. Two water taps together can fill a tank in $9\frac{3}{8}$ hours. The tap of larger diameter takes 10 hours less than the smaller one to fill the tank separately. Find the time in which each tap can separately fill the tank.

OR

Solve for x: $\frac{3}{x+1} + \frac{4}{x-1} = \frac{29}{4x-1}; x \neq 1, \frac{1}{4}$.

36. In an equilateral triangle ABC, D is a point on side BC such that $BD = \frac{1}{3}BC$. Prove that $9\,AD^2 = 7AB^2$.

37. Construct a right triangle in which sides (other than the hypotenuse) are 8 cm and 6 cm. Then construct another triangle whose sides are $\frac{3}{5}$ times the corresponding sides of the right triangle.

38. If the angle of a cloud from a point h metres above a lake is α and the angle of depression of its reflection in the lake is β, prove that the height of the cloud is $\frac{h(tan\beta+tan\alpha)}{tan\beta-tan\alpha}$.

OR

The angle of elevation of an aeroplane from a point A on the ground is 60^0. After a flight of 30 seconds, the angle of elevation changes to 30^0. If the aeroplane is flying at a constant height of $3600\sqrt{3}$ metres then find the speed of the aeroplane.

39. A container shaped like a right circular cylinder having diameter 12 cm and height 15 cm is full of ice cream. The ice cream is to be filled into cones of height 12 cm and diameter 6 cm, having a hemispherical shape on the top. Find the number of such cones which can be filled with ice cream.

OR

A metallic right circular cone 20 cm high whose vertical angle is 60^0 which is cut into two parts at the middle of its height by a plane parallel to its base. If the frustum so obtained be drawn into a wire of diameter $\frac{1}{16}$ cm, find the length of the wire.

40. The following table gives the height of 40 trees in meters:

Height (in meter)	0-8	8-16	16-24	24-32	32-40	40-48
No. of trees	3	7	13	7	8	2

Draw the less than type and a more than type ogive from the given data. Hence find the median value.

KENDRIYA VIDYALAYA SANGATHAN
STANDARD (041)

2019-20
Answer Key

Class- X   Time-3Hours

Subject- Mathematics

Marks: 80

Question No.	Answer	Question No.	Answer	Question No.	Answer	Question No.	Answer
1.	d	**11.**	c	**21.**	Correct Proof	**31.**	X=6, y= 2 Or Correct Proof
2.	c	**12.**	a	**22.**	k≠6	**32.**	Correct Proof
3.	c	**13.**	d	**23.**	K=14	**33.**	51.33 sq.cm
4.	c	**14.**	d	**24.**	38 Or Yes, 75	**34.**	1/2, 2/25, 2/25
5.	b	**15.**	d	**25.**	2 :3 Or -9 & 3	**35.**	15 Hours, 25 Hours Or x= -7, 4
6.	b	**16.**	30^0	**26.**	3/4	**36.**	Correct Proof
7.	c	**17.**	3	**27.**	Correct Proof	**37.**	Correct Construction
8.	a	**18.**	$\frac{1}{3}\pi h(r_1^2 + r_2^2 + r_1 r_2)$	**28.**	x=4, y= 5 or a=5, b=2	**38.**	Correct Proof Or 240 mt./sec
9.	d	**19.**	Corresponding Sides	**29.**	Correct Proof Or Correct Proof	**39.**	10 Or 7964.444 Mt.
10.	d	**20.**	Zero	**30.**	4	**40.**	Correct ogive

KENDRIYA VIDYALAYA SANGATHAN

BLUE PRINT OF MODEL QUESTION PAPER 2

2019-2020

MATHEMATICS- Standard (041)

CLASS:-X

Time Allowed: 03Hours   **Maximum Marks: 80**

S.NO	UNITS	VSA	SA-I	SA-II	LA	TOTAL MARKS
1	**Number Systems**	1(1)	2(1)	3 (1)	-	6(3)
2	Algebra	1(1)	4(2)	3(1)	12(3)	20(7)
3	Coordinate Geometry	1(1)	2(1)	3(1)	-	6(3)
4	Geometry	2(2)	2(1)	3(1)	8(2)	15(6)
5	Trigonometry	-	2(1)	6 (2)	4(1)	12(4)
6	Mensuration	-	-	6(2)	4(1)	10(3)
7	Statistics and Probability	1(1)	-	6(2)	4(1)	11(4)
	TOTAL	6(6)	12(6)	30(10)	32(8)	80(30)

- NO OF QUESTIONS ARE GIVEN IN THE BRACKET

KENDRIYA VIDYALAYA SANGATHAN

MODEL QUESTION PAPER - 2

2019-20

Mathematics- STANDARD (041)

Class:-X   Time: 3 Hrs

                                                                                                  Marks: 80

General Instructions:
   (i)   All questions are compulsory.
   (ii)   The question paper comprises of five sections, A, B, C, D and E. You are to attempt all the sections.
   (iii)   Section A comprises of 10 multiple choice questions (MCQs) one mark each. Section B comprises of 10 questions of one mark each. Section C comprises of 6 questions of 2 marks each. Section D comprises of 8 questions of 3 marks each. Section E comprises of 6 questions of 4 marks each.
   (iv)   There is no overall choice. However, an internal choice has been provided in Section C,D and E.
   (v)   Use of calculator and other electronic device is not permitted.

SECTION – A

Question Numbers 1 to 10 carry one mark each.

1. How many rational number are there between two rational number
   a) one   b) two   c) three   d) infinite

2. The sum and product of the zeroes of a quadratics polynomial are 2 and -15 respectively the quadratic polynomial is
   a) x^2-2x+ 5   b) x^2 -2x-15   c) x^2 +2x-15   d) x^2 -2x+15

3. For what value of k , the pair of linear equations 2x-y-3=0 and 2kx +y -2= 0 has 103olution = 1 and y= -1
   a) 3   b) 3/2   c) 4   d) -4

4. If the point (a ,0) , (0 , b) and (1 , 1) are collinear then 1/a +1/b is
   a) 1   b) 2   c) 0   d) 3

5. If sin A= 1/2 then angle A is
   a) 30^0 b) 60^0 c) 45^0 d) 90^0

6. The sum of first five natural number is
   a) 10   b) 12   c) 15   d) 20

7. The ratio of radii of two sphere is 4:3. The ratio of their volume is
   a) 64:27   b) 27:64   c) 16:9   d) 9:16

8. The probability of getting a even number when die is thrown is
   a) 1/4   b) 3/4   c) 1/2   d)3/5

9. The mode of the observation 2, 3, 6, 7, 2, 8 ,2 is
   a) 3   b) 2   c) 4   d) 6

10. The distance of the point (3, 4) from origin is
    a) 5   b) 6   c) 10   d) 12

SECTION – B
Question Numbers 11 to 20 carry one mark each.

The following questions (11 to 15) consist of two statements- Assertion (A) and Reason(R).

Answer the questions selecting the appropriate option given below:

a) Both A and R are true and R is the correct explanation for A

b) Both A and R are true and R is not correct explanation for A

c) A is true but R is false

d) A is false but R is true

11. **Assertion (A) :** The probability of getting a red card from a pack , when a card is drawn at random, is 1/4
    Reason (R): A pack of card contain 26 red and 26 black cards.

12. **Assertion (A) :** Area of right isosceles triangle with one of the equal side of length 12cm is 72cm^2
    Reason (R): Area of triangle = 1/2 x base x height

13. **Assertion (A) :** The fourth angle of a quadrilateral with two right angles , and the measure of angles are 80^0 and 100^0
    Reason (R): The sum of angles of a quadrilateral is 360^0

14. **Assertion (A) :** The points (a/3 , a) lies on x= 3y
    Reason (R): Every point on the graph of a linear equation in two variables is a solution of linear equation.

15. **Assertion (A):** 3x^3 -2 is a binomial.
    Reason (R): The degree of a cubic polynomial is 3.

Fill in blanks. (Q. No. 16 to 20)

16. If x-1 is a factor of 3x^3 + 2x^2 -3x +k, then the value of k is....................

17. Diagonals of a rectangle.............. each other and are equal.

18. A geometrical construction is the process of drawing a geometrical figure with the un graduated ruler and a.....................

19. The volume of a sphere with diameter 6cm, in terms of π , is.................

20. In a frequency distribution, the mid value of a class is 120 and width is 20. The upper limit of the class is....................

SECTION – C
(Question numbers 21 to 26 carry 2 marks each)

Q.21: Find the roots of x^2 - 5x +6 = 0

Q.22: Write the condition to be satisfied by q so that a rational number p/q has a terminating

decimal expansion. Give example.

Q.23: If the mid-point of a segment joining A $(\frac{x}{2}, \frac{y+1}{2})$ and B (X+1 ,Y-3) IS C (5 , - 2) find x and y .

OR

Find the distance between the points (-7/5, 2) and (3/5, 3)

Q.24: If sin A = $\frac{1}{3x}$ and tan A = $\frac{x}{3}$ Then find the value of $(x^2 - \frac{1}{x^2})$

Q.25: Do the equation 5x+7y=8 and 10x+14y = 4 represent a pair of coincident lines? Justify your answer.

Q.26: If the system of equation 4x+y = 3 and (2k-1) x + (k-1) y = 2k+1 is inconsistent, then find k.

SECTION – D
(Question numbers 27 to 34 carry 3 marks each)

Q.27: Prove that $\sqrt{7}$ is irrational number.

Q28: Draw the graph of the linear equation x-y = -1 and 3x+2y -1= 0.

Q29: In an AP the first term is 2, the last term is 29 and the sum of the terms is 155, find the common difference of AP.

Q.30: IF B (x, y) is a point on the line segment joining the point A (a, b) and C (b, a), then prove that x + y = a + b

Q.31: Prove that $(\frac{1+\tan^2 A}{1+\cot^2 A}) = (\frac{1-\tan A}{1-\cot A})^2 = \tan^2 A$

Q.32: In a fig, AB and CD are two diameters of a circle (with centre O) perpendicular to each other and OD is the diameter of smaller circle. If OA =7cm. find the area of shaded region.
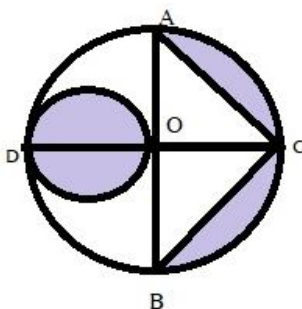
Q.33: Which term of the AP 121, 117, 113 .... is its first negative term?

Q.34: Solve the following equation

2x+3y -13 = 0

5x -4y + 2 = 0

SECTION – E
(Question numbers 35 to 40 carry 4 marks each)

Q.35: Reshma wishes to fit three rods in the shape of a right triangle. The hypotenuse is to be 2cm longer

than The base and 4cm longer than the altitude. What should be the length of rods?

OR

Solve for x: $\frac{2}{x+1} + \frac{3}{2(x-2)} = \frac{23}{5x}$, x ≠ 0 , -1 , 2

Q.36: Draw a right triangle ABC in which AC= AB= 4cm and ∠A = 60^0. Draw a triangle similar to ΔABC with its sides
equal   to $\frac{5}{4}$ of the corresponding sides of ΔABC.

Q.37: Prove that the ratio of the areas of two similar triangles is equal to the square of the ratio of their

Corresponding sides.

Q.38: Find the missing frequencies in the following frequency distribution if it is known that the mean of the

distribution is 62.8 and sum of all the frequency is 50.

Class	0 – 20	20 - 40	40 - 60	60 – 80	80 – 100	100 – 120
Frequency	5	F_1	10	F_2	7	8

Q.39: From a solid circular cylinder with height 10 cm and radius of the base 6 cm , a right circular cone of the

same height and same base is removed .find the volume of the remaining solid. Also calculate the whole

Surface area.

Q.40: The angle of elevation of the top 'A 'of a vertical tower AB from a point 'P' on the ground is 60^0.

At a point 'Q' 40 m vertically above 'P', the angle of elevation is 45^0. Find the height of the tower AB

and the distance PA.

OR

An aeroplane when flying at a height of 4000 m from the ground passes vertically above another

aeroplane at an instant when the angles of the elevation of the two planes from the same point on

The ground are 60^0 and 45^0, respectively. Find the vertical distance between the aero planes at that

Instant.

Class- X   Time-3Hours

Subject- Mathematics

Marks: 80

Question No.	Answer	Question No.	Answer	Question No.	Answer	Question No.	Answer
1.	d	**11.**	d	**21.**	x=3,2	**31.**	Correct Proof
2.	b	**12.**	a	**22.**	Factor 2 and 5	**32.**	66.5 cm^2
3.	b	**13.**	a	**23.**	x=6 & y= -1 or $\sqrt{5}$ Unit	**33.**	32nd term
4.	a	**14.**	d	**24.**	1/9	**34.**	x= 2 & y= 3
5.	a	**15.**	d	**25.**	No	**35.**	8, 6 & 10 cm Or 4, -23/11
6.	c	**16.**	k=-2	**26.**	3/2	**36.**	Right construction
7.	a	**17.**	Bisect	**27.**	Correct Proof	**37.**	Correct Proof
8.	c	**18.**	Compass	**28.**	Correct Graph	**38.**	F$_1$=8, F$_2$ =12
9.	b	**19.**	36π	**29.**	3	**39.**	Volume= 240 π cm^3 Surface area= (156 + 12$\sqrt{34}$
10.	a	**20.**	100	**30.**	Correct Proof	**40.**	Height of tower= 54.64 & PA = 109.3 mt. Or $4000(\dfrac{\sqrt{3}-1}{\sqrt{3}})$

MATHEMATICS-Basic
Code (241)

CLASS - X
Blue Print of Question Paper- 3

Topic/Unit	Section -A	Section -B	Section –C	Section -D	Total Marks
Number System	1(1)	2(1)	3(1)	-	06
Algebra	6(6)	4(2)	6(2)	4(1)	20
Co-Geometry	1(1)	2(1)	3(1)	-	06
Geometry	4(4)	-	3(1)	8(2)	15
Trigonometry	3(3)	2(1)	3(1)	4(1)	12
Mensuration	3(3)	-	3(1)	4(1)	10
Statistics & Probability	2(2)	2(1)	3(1)	4(1)	11
Total	**20(20)**	**12(6)**	**24(8)**	**24(6)**	**80**

Note: Number of questions are given within brackets and marks outside the brackets

MATHEMATICS-Basic
Code (241)
Class-X
Model Question Paper- 3

Time allowed: 3 hours   **Maximum Marks: 80**

General Instructions:
1. *All questions are compulsory.*
2. *The question paper comprises of five sections, A, B, C, D and E. You are to attempt all the sections.*
3. *Section A comprises of 10 multiple choice questions (MCQs) one mark each. Section B comprises of 10 questions of one mark each. Section C comprises of 6 questions of 2 marks each. Section D comprises of 8 questions of 3 marks each. Section E comprises of 6 questions of 4 marks each.*
4. *There is no overall choice. However, an internal choice has been provided in Section C, D and E.*
5. *Use of calculator and other electronic device is not permitted.*

SECTION – A
Question Numbers 1 to 10 carry one mark each.

1. The value of k, so that quadratic equation $2x^2 + kx + 3 = 0$ has two equal roots.
   (a) $2\sqrt{6}$ (b) $\pm 2\sqrt{6}$ (c) $\pm 2\sqrt{3}$ (d) none of these

2. If $P(E) = 0.05$, what is the probability of 'not E'?
   (a) 0.95   (b) 0.05   (c) 1.05   (d) 1

3. In the figure, if angle ATO = 40°, find angle AOB.
   (a) 120°   (b) 90°   (c) 150°   (d) 100°
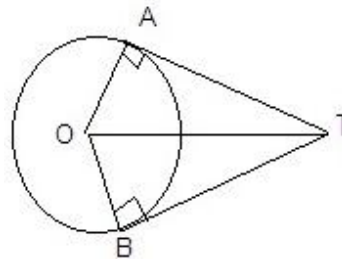
4. Decimal expansion of $\frac{23}{2^3 5^2}$ will be
   (a) Terminating   (b) non- terminating   (c) Non- terminating & repeating   (d) non- terminating & non- repeating

5. If the sum of n terms of an AP is $3n^2 + 5n$ then which term is 164?
   (a) 26th   (b) 27th   (c) 28th   (d) none of these

6. If radii of two concentric circles are 6 cm and 10 cm, then length of the chord of one circle which is tangent to the other circle is
   (a) 5 cm   (b) 8 cm   (c) 12 cm   (d) 16 cm

7. If $\sin\alpha = \frac{1}{2}$ and $\cos\beta = \frac{1}{2}$, then $\alpha + \beta$ is
   (a) 0°   (b) 30°   (c) 60°   (d) 90°

8. The area of the triangle formed by the line $\frac{x}{a} + \frac{y}{b} = 1$ with the co-ordinate axes is

   (a) ab   (b) 2ab   (c) $\frac{1}{2}$ab   (d) $\frac{1}{4}$ab

9. The sum areas of a minor sector and the corresponding major sector of a circle is equal to

   (a) Area of the circle   (b) $\frac{1}{2}$ Area of the circle   (c) $\frac{1}{4}$ Area of the circle   (d) $\frac{3}{4}$ Area of the circle

10. The pair of equations x = 0 , y =0 represents

   (a) Parallel lines   (b) coincident lines   (c) perpendicular lines (d) non-intersecting lines

SECTION – B
Question Numbers 11 to 20 carry one mark each.

State whether the following statements are true or false. (Q No. 11 to 15)

11. A polynomial cannot have more than one zero.

12. If a line divides any two sides of a triangle in same ratio the it is parallel to the third side.

13. In ΔABC $\sin\left(\frac{B+C}{2}\right) = \cos\frac{A}{2}$

14. If the graph of a polynomial intersects x-axis at one point then it is a quadratic polynomial.

15. Mode + 2 mean = 3 median

Fill in the Blanks. (Q No. 16 to 20)

16. A tangent to a circle intersects it in.................... point(s).

17. The volume of frustum of a cone whose height is h and radius of bases be r_1 and r_2 is...........

18. The total surface area of a solid hemisphere having radius R is...............

19. cosec $(90 - \theta)$ =............

20. The sum of first n natural number is......................

SECTION – C
(Question numbers 21 to 26 carry 2 marks each)

21. Given that HCF (306, 657) = 9, Find LCM (306, 657).

22. Evaluate $\frac{\cos\;\;°}{\sec\;\;°+cosec\;\;°}$

23. Find a quadratic polynomial whose sum and product of zeroes are $\frac{1}{4}$ and -1 respectively.

24. Which term of the AP: 3, 8, 13, 18......... is 78?

OR

Find the 20th term from the last term of the AP: 3, 8, 13……………253.

25. If the points (1, 2), (4, y), (x, 6) and 3, 5) are the vertices of a parallelogram taken in order, find x and y.

26. Savita and Hamida are friends. What is the probability that both will have (a) different birthdays? (b) the same birthday ? (ignoring a leap year)

SECTION – D
(Question numbers 27 to 34 carry 3 marks each)

27. Use Euclid's division algorithm to find the HCF of 4052 and 12576.
28. Solve $6x + 3y = 6xy$, $2x + 4y = 5xy$

OR

For what values of k will the following pair of linear equations have infinitely many solutions?

$Kx + 3y – (K-3) = 0$ $12x + ky – k = 0$

29. Find the sum of the first 15 multiples of 8.
30. Find the ratio in which the line segment joining A (1, -5) and B (-4, 5) is divided by the x-axis. Also find the coordinates of the point of division.

OR

Find the area of the quadrilateral whose vertices, taken in order are (-4, -2), (-3, -5), (3, -2) and (2, 3)

31. Prove that the length of tangents drawn from an external point to a circle are equal.
32. Evaluate : $2\left(\dfrac{\cos\ ^\circ}{\sin\ ^\circ}\right) - \sqrt{3}\left(\dfrac{\cos\ ^\circ cosec\ ^\circ}{\tan\ ^\circ \tan\ ^\circ \tan\ ^\circ}\right)$

OR

Prove that $(1 + \tan\theta + \sec\theta)(1 + \cot\theta – cosec\theta) = 2$

33. OACB is a quadrant of a circle with centre O and radius 3.5 cm. If OD = 2 cm, find the area of the
    (i) quadrant OACB
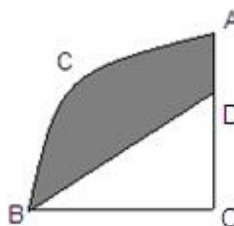
    (ii) shaded region
34. A bag contains 7green, 10 blue and 5 red balls. A ball is drawn at random. Find the probability of this ball being a :
    a. Blue ball.
    b. red ball or a green ball
    c. not a green ball

SECTION – E
(Question numbers 35 to 40 carry 4 marks each)

35. Two water tapes together can fill a tank in $9\dfrac{3}{8}$ hours. The tap of larger diameter takes 10 hours less than the smaller one to fill the tank separately. Find the time in which each tap can separately fill the tank.

OR

In a class test, the sum of Shefali's, marks in Mathematics and English is 30. Had she got 2 marks more in Mathematics and 3 marks less in English, the product of their marks would have been 210. Find the marks in the two subjects.

36. Prove that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides. Let ΔABC ~ΔDEF and their areas be 64 cm² and 121 cm² respectively. If EF = 15.4 cm, Find BC.

37. Draw a triangle ABC with side BC = 7 cm, ∠B =45°, ∠A = 105° . Then construct a triangle whose sides are $\frac{4}{3}$ times the corresponding sides of ΔABC. (Also write steps of construction)

38. The angle of elevation of an aeroplane from a point on the ground is 60° . After a flight of 30 second the angle of elevation becomes 30°. If the aeroplane is flying at a constant height of $3000\sqrt{3}$ m, find the speed of theaeroplane.

OR

From the top of a 7 m high building, the angle of elevation of the top of a cable tower is 60° and the angle of depression of its foot is 45°. Determine the height of the tower.

39. A sphere, of diameter 12 cm, is dropped in a right circular cylindrical vessel, partly filled with water. If the sphere is completely submerged in water, the water level in the cylindrical vessel rises by $3\frac{5}{9}$ cm. Find the diameter of the cylindrical vessel.

OR

From a solid cylinder whose height is 2.4 cm and diameter 1.4 cm, a conical cavity of the same height and same diameter is hollowed out. Find the total surface area of the remaining solid to the nearest cm².

40. Find the values of the frequencies x and y in the following frequency distribution table if median is 32.

Marks	0-10	10-20	20-30	30-40	40-50	50-60	Total
No .of Students	10	X	25	30	Y	10	100

MATHEMATICS-Basic
Code (241)
Class-X
Model Question Paper- 3

Question No.	Answer	Question No.	Answer	Question No.	Answer	Question No.	Answer
1	b	11.	False	21	22338	31	proof
2	a	12	True	22	$\frac{3\sqrt{2}}{8} - \frac{\sqrt{6}}{8}$	32	1
3	d	13	True	23	$4x^2-x-4$	33	(i)$9\frac{5}{8}$ cm^2 (ii)$6\frac{1}{8}$ cm^2
4	a	14	False	24	16th or 158	34	(i)P(blue ball) $=\frac{5}{11}$ (ii)P(red or a green ball) = $\frac{6}{11}$ (iii)P(not a green ball) = $\frac{15}{22}$
5	b	15	True	25	X = 6, y= 3	35	25 h & 15 h OR Math=12, English =18 or math's=13, English =17
6	d	16	one	26	(i) $\frac{364}{365}$ (ii) $\frac{1}{365}$	36	11.2 cm
7	d	17.	$\frac{1}{3}\pi h(r_1^2 + r_2^2 + r_1r_2)$	27	4	37	construction
8	c	18	$3\pi R^2$	28	X= 1 ,y =2 Or K= 6	38	720 km/h OR $7(\sqrt{3} + 1)$ m
9	a	19	$\sec\theta$	29	960	39	18 cm OR 18 cm^2
10	c	20	$\frac{n(n+1)}{2}$	30	1:1 & (-3/2,0) OR 28 q. unit	40	X= 9, y=16

KENDRIYA VIDYALAYA SANGATHAN
MATHEMATICS-Basic
Code (241)
Class-X
Model Question Paper- 4

Time allowed: 3 hours   **Maximum Marks: 80**

General Instructions:
1. *All questions are compulsory.*
2. *The question paper comprises of five sections, A, B, C, D and E. You are to attempt all the sections.*
3. *Section A comprises of 10 multiple choice questions (MCQs) one mark each. Section B comprises of 10 questions of one mark each. Section C comprises of 6 questions of 2 marks each. Section D comprises of 8 questions of 3 marks each. Section E comprises of 6 questions of 4 marks each.*
4. *There is no overall choice. However, an internal choice has been provided in Section C, D and E.*
5. *Use of calculator and other electronic device is not permitted.*

SECTION – A
Question Numbers 1 to 10 carry one mark each.

1. In an AP if a =-7.2, d =3.6,a_n =7.2, then n is:
    (A) 1   (B) 3   (C) 4   (D) 5

2. A tangent PQ at a point P of a circle of radius 5cm meets a line through the centre O. At a point Q so that OQ = 12cm, length PQ is:
    A) 12 cm   B) 13cm   C) 8.5cm   D) $\sqrt{119}$ cm

3. The radii of the ends of a frustum of a cone of height h cm are r_1 cm and $r_2$cm respectively. The volume in cm^3 of the frustrum of the cone is
    a. $\frac{1}{3}\pi h(r_1^2 + r_2^2 + r_1 r_2)$ b. $\frac{1}{3}\pi h(r_1^2 + r_2^2 - r_1 r_2)$
    c. $\frac{1}{3}\pi h(r_1^2 - r_2^2 + r_1 r_2)$ d. $\frac{1}{3}\pi h(r_1^2 - r_2^2 - r_1 r_2)$

4. When a dice is thrown, the probability of getting an odd number less than 3 is
    (A)$\frac{1}{6}$ (B)$\frac{1}{3}$ (C) $\frac{1}{2}$ (D)0

5. If cos A = $\frac{4}{5}$, then the value of tan A is ;
    (A)$\frac{3}{5}$ (B)$\frac{3}{4}$ (C) $\frac{4}{3}$ (D)$\frac{5}{3}$

6. The product of zeroes of x^2 +4x-5=0
    A) 5   B)-5   C) 0   D)4

7. A ladder makes an angle of 60˚ with the ground when placed against a wall. If the foot of the ladder is 2m away from the wall, then the length of the ladder is
    A) 4m   B) $\sqrt{3}$m   C) 2$\sqrt{2}$m   D) 5$\sqrt{3}$m

8. The curved surface area of a right circular cone of height 15 cm and the base diameter16cm is
    A) 60πcm^2 B) 120πcm^2 C) 136πcm^2 D) none of these

9. The quadratic equation $2x^2 - \sqrt{5}\,x + 1 = 0$ has

    A) Two distinct roots   B) Two equal roots   C) No real roots   D) More than 2 two real roots

10. The area of a triangle with vertices A(3,0),B(7,0) and C(8.4)

        A) 6   B) 8   C) 14   D)28

SECTION – B

Question Numbers 11 to 20 carry one mark each.

State whether the following statements are true or false. (Q. No. 11 to 15)

    11. Every fraction is a rational number

    12. All congruent triangles are similar

    13. The probability of event can be greater than 1.

    14. A pair of intersecting lines representing a pair of linear equation in two variable has a unique solution

   **15.** 17/30 is a terminating decimal

Fill in the blanks. (Q. No. 16 to 20)

    16. If the probability of E is 1/3, the probability of "not E" is ………………

    17. A line intersecting circle in two points is called a ……………

    18. The sum of first 10 terms of the AP 10,7,4…... is………

    19. A quadratic equation cannot have more than …………roots

    20. $5 + \sqrt{3}$ is a/an………….. number

SECTION – C
(Question numbers 21 to 26 carry 2 marks each)

21. Find the area of the shaded region in the given figure, if ABCD is a square of side 14 cm and APD and BPC are semicircles.
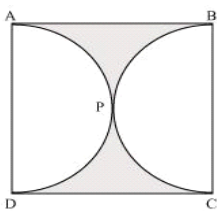
22. In Fig. DE || AC and DF || AE. Prove that

$$\frac{BF}{FE} = \frac{BE}{EC}$$
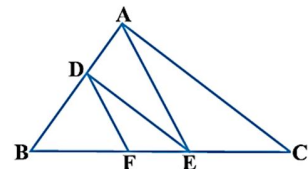
23. For what value of pare 2p+1, 13, 5p-3, three consecutive terms of and A.P.?

24. What is the distance between the points A (5, 2) and B (2,-2).

25. A card is drawn at random from a well shuffled deck of 52 cards. Find the probability of getting:

    (i) a king   (ii) a king of red suit

26. Use Euclid`s division algorithm to find the HCF of 867 and 255.

SECTION – D
(Question numbers 27 to 34 carry 3 marks each)

27.   Prove that the parallelogram circumscribing a circle is a rhombus.

28.   A fraction becomes $\frac{1}{3}$ when 1 is subtracted from the numerator and it becomes $\frac{1}{4}$ when 8 is added to its denominator. Find the fraction.

<div align="center">OR</div>

A part of monthly hostel charges is fixed and the remaining depends on the number of days one has taken food in the mess. When a student A takes food for 20 days she has to pay 1000 as hostel charges whereas a student B, who takes food for 26 days, pays ` 1180 as hostel charges. Find the fixed charges and the cost of food per day.

29.   The sum of 5th and 9th terms of an A.P. is 72 and the sum of 7th and 12th terms is 97. Find the A.P.

30. In Fig., XY and X′Y′ are two parallel tangents to a circle with centre O andanother tangent AB with point of
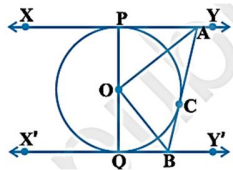
contact C intersecting XY at A and X′Y′ at B. Prove that ∠ AOB = 90°.

31.   In fig. 10, PQ=24cm, PR=7cm and O is the centre of the circle. Find the area of the shaded region. (Take π=3.14).
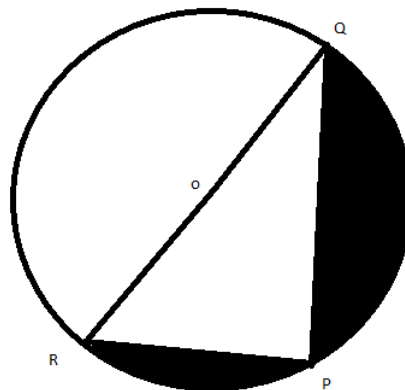

FIG. 10

32. The king, queen and jack of clubs are removed from a deck of 52 playing cards and remaining cards are shuffled. A card is drawn from the remaining cards. Find the probability of getting a card of
   (i) heart   (ii) queen   (iii) clubs.

33. Prove that.

$$(\sin A + \text{cosec } A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A$$

34. Metallic spheres of radii 6 cm, 8 cm and 10 cm, respectively, are melted to form a single solid sphere. Find the radius of the resulting sphere.

SECTION – E
(Question numbers 35 to 40 carry 4 marks each)

35. Construct a triangle ABC in which BC=6.5cm, AB=4.5cm and $\angle ABC = 60^0$. Construct a triangle similar to this triangle whose sides are $\frac{3}{4}$ of the corresponding sides of the triangle ABC.

36. A train travels 360 km at a uniform speed. If the speed had been 5 km/h more, it would have taken 1 hour less for the same journey. Find the speed of the train.

OR

The sum of the reciprocals of Rehman's ages, (in years) 3 years ago and 5 years from now is $\frac{1}{3}$. Find his present age.

37. Prove that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

OR

In an equilateral triangle ABC, D is a point on side BC such that BD $=\frac{1}{3}$BC. Prove that 9 AD^2 = 7AB^2.

38. From a point on the ground, the angles of elevation of the bottom and top of a transmission tower fixed at the top of a 20 m high building are 45° and 60° respectively. Find height of the tower.

OR

Two poles of equal heights are standing opposite each other on either side of the road, which is 80 m wide. From a point between them on the road, the angles of elevation of the top of the poles are 60° and 30°, respectively. Find the height of the poles and the distances of the point from the Poles.

39. A drinking glass is in the shape of a frustum of acone of height 14 cm. The diameters of its two circular ends are 4 cm and 2 cm. Find the capacity of the glass.

40. The median of the following data is 525. Find the values of x and y, if the total frequency is 100.

Class interval	Frequency
0 – 100	2
100 – 200	5
200 – 300	x
300 – 400	12
400 – 500	17
500 – 600	20
600 – 700	y
700 – 800	9
800 – 900	7
900 – 1000	4

MATHEMATICS-Basic
Code (241)
Class-X
Model Question Paper- 4

Question No.	Answer	Question No.	Answer	Question No.	Answer	Question No.	Answer
1.	D	11.	T	21.	Area = 42 cm^2	31.	Area = 4523/28 cm^2 = 161.53 cm^2
2.	D	12.	T	22.	Correct proof	32.	13/49, 3/49, 10/49
3.	A	13.	F	23.	P = 4	33.	Correct steps of proof
4.	A	14.	T	24.	5 unit	34.	Radius = 12cm
5.	C	15.	F	25.	1/13,1/26	35.	Correct construction
6.	B	16.	2/3	26.	HCF =51	36.	Speed = 40 km/h or present age = 7years
7.	A	17.	SECANT	27.	Correct proof	37.	Statement, Fig., proof or correct steps of proof
8.	C	18.	-35	28.	Fraction = 5/12 OR Fixed charge = Rs. 400 and per day charge Rs. 30	38.	Height of tower = 20 ($\sqrt{3}$-1) m or Height = 20$\sqrt{3}$ m, Distance 20 m and 60 m
9.	C	19.	2	29.	a = 6, d = 5, A. P. : 6, 11, 16, ….	39.	Capacity = $102\frac{2}{3}$ cm^3
10.	B	20.	IRRATIONAL	30.	Correct proof	40.	x = 9, y = 15

✱✱✱

Series JMS/2

रोल नं.
Roll No.

परीक्षार्थी कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें ।

Candidates must write the Code on the title page of the answer-book.

- कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ **11** हैं ।
- प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए कोड नम्बर को छात्र उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें ।
- कृपया जाँच कर लें कि इस प्रश्न-पत्र में **30** प्रश्न हैं ।
- **कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, प्रश्न का क्रमांक अवश्य लिखें ।**
- इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है । प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा । 10.15 बजे से 10.30 बजे तक छात्र केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे ।
- Please check that this question paper contains **11** printed pages.
- Code number given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
- Please check that this question paper contains **30** questions.
- **Please write down the Serial Number of the question before attempting it.**
- 15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer-book during this period.

गणित
MATHEMATICS

निर्धारित समय : 3 घण्टे
Time allowed : 3 hours

अधिकतम अंक : 80
Maximum Marks : 80

सामान्य निर्देश :

(i) **सभी** प्रश्न अनिवार्य हैं ।

(ii) इस प्रश्न-पत्र में **30** प्रश्न हैं जो चार खण्डों — अ, ब, स और द में विभाजित हैं ।

(iii) खण्ड अ में **एक-एक** अंक वाले **6** प्रश्न हैं । खण्ड ब में **6** प्रश्न हैं जिनमें से प्रत्येक **2** अंक का है । खण्ड स में **10** प्रश्न **तीन-तीन** अंकों के हैं । खण्ड द में **8** प्रश्न हैं जिनमें से प्रत्येक **4** अंक का है ।

(iv) प्रश्न-पत्र में कोई समग्र विकल्प नहीं है । तथापि **1** अंक वाले **2** प्रश्नों में, **2** अंकों वाले **2** प्रश्नों में, **3** अंकों वाले **4** प्रश्नों में और **4** अंकों वाले **3** प्रश्नों में आंतरिक विकल्प प्रदान किए गए हैं । ऐसे प्रश्नों में आपको दिए गए विकल्पों में से केवल **एक** प्रश्न ही करना है ।

(v) कैलकुलेटरों के प्रयोग की अनुमति **नहीं** है ।

General Instructions :

(i) **All** questions are compulsory.

(ii) The question paper consists of **30** questions divided into four sections — A, B, C and D.

(iii) Section A contains **6** questions of **1** mark each. Section B contains **6** questions of **2** marks each, Section C contains **10** questions of **3** marks each and Section D contains **8** questions of **4** marks each.

(iv) There is no overall choice. However, an internal choice has been provided in **two** questions of **1** mark each, **two** questions of **2** marks each, **four** questions of **3** marks each and **three** questions of **4** marks each. You have to attempt only **one** of the alternatives in all such questions.

(v) Use of calculators is **not** permitted.

<div align="center">

खण्ड अ

SECTION A

</div>

प्रश्न संख्या 1 से 6 तक प्रत्येक प्रश्न 1 अंक का है ।

Question numbers 1 to 6 carry 1 mark each.

1. यदि म.स. (HCF) (336, 54) = 6 है, तो ल.स. (LCM) (336, 54) ज्ञात कीजिए ।

   If HCF (336, 54) = 6, find LCM (336, 54).

2. द्विघात समीकरण $2x^2 - 4x + 3 = 0$ के मूलों की प्रकृति ज्ञात कीजिए ।

   Find the nature of roots of the quadratic equation $2x^2 - 4x + 3 = 0$.

3. समान्तर श्रेढ़ी $\frac{1}{a}, \frac{3-a}{3a}, \frac{3-2a}{3a}, \dots$ $(a \neq 0)$ के लिए सार्व अन्तर ज्ञात कीजिए ।

   Find the common difference of the Arithmetic Progression (A.P.)

   $$\frac{1}{a}, \frac{3-a}{3a}, \frac{3-2a}{3a}, \dots \ (a \neq 0)$$

4. मान ज्ञात कीजिए :

   $$\sin^2 60° + 2 \tan 45° - \cos^2 30°$$

<div align="center">

अथवा

</div>

यदि $\sin A = \dfrac{3}{4}$ है, तो sec A परिकलित कीजिए ।

Evaluate :

$$\sin^2 60° + 2 \tan 45° - \cos^2 30°$$
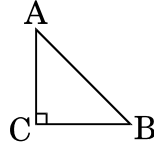
<div align="center">**OR**</div>

If $\sin A = \dfrac{3}{4}$, calculate sec A.

5. x-अक्ष पर स्थित बिन्दु P के निर्देशांक लिखिए जो बिन्दु A(– 2, 0) तथा बिन्दु B(6, 0) से समान दूरी पर हो ।

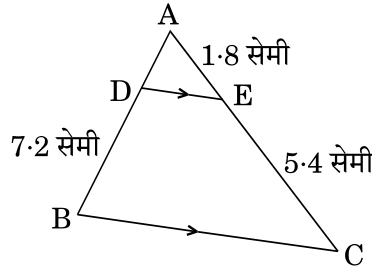Write the coordinates of a point P on x-axis which is equidistant from the points A(– 2, 0) and B(6, 0).

6. आकृति 1 में, ABC एक समद्विबाहु त्रिभुज है जिसका कोण C समकोण है तथा AC = 4 cm है । AB की लंबाई ज्ञात कीजिए ।

<div align="center">*आकृति 1*</div>

<div align="center">**अथवा**</div>

आकृति 2 में, DE ǁ BC है । भुजा AD की लम्बाई ज्ञात कीजिए जबकि दिया गया है AE = 1·8 सेमी, BD = 7·2 सेमी तथा CE = 5·4 सेमी ।

<div align="center">*आकृति 2*</div>

In Figure 1, ABC is an isosceles triangle right angled at C with AC = 4 cm. Find the length of AB.
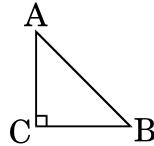

<div align="center">*Figure 1*</div>

<div align="center">**OR**</div>

In Figure 2, DE ∥ BC. Find the length of side AD, given that AE = 1·8 cm, BD = 7·2 cm and CE = 5·4 cm.
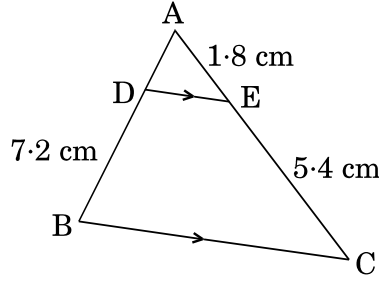

Figure 2

खण्ड ब
SECTION B

प्रश्न संख्या 7 से 12 तक प्रत्येक प्रश्न के 2 अंक हैं ।
Question numbers 7 to 12 carry 2 marks each.

7. वह सबसे छोटी संख्या लिखिए जो 306 तथा 657 दोनों से पूर्णतया विभाजित हो ।

Write the smallest number which is divisible by both 306 and 657.

8. x और y में एक संबंध ज्ञात कीजिए ताकि बिन्दु A(x, y), B(–4, 6) तथा C(–2, 3) संरेखीय हों ।

अथवा

उस त्रिभुज का क्षेत्रफल ज्ञात कीजिए जिसके शीर्ष (1, – 1) (– 4, 6) तथा (– 3, – 5) हैं ।

Find a relation between x and y if the points A(x, y), B(– 4, 6) and C(– 2, 3) are collinear.

OR

Find the area of a triangle whose vertices are given as (1, – 1) (– 4, 6) and (– 3, – 5).

9. एक जार में केवल नीले, काले तथा हरे कंचे हैं । इस जार में से यादृच्छया एक नीले कंचे के निकालने की प्रायिकता $\frac{1}{5}$ है तथा उसी जार में से एक काले कंचे के यादृच्छया निकालने की प्रायिकता $\frac{1}{4}$ है । यदि जार में 11 हरे रंग के कंचे हैं, तो जार में कुल कंचों की संख्या ज्ञात कीजिए ।

The probability of selecting a blue marble at random from a jar that contains only blue, black and green marbles is $\frac{1}{5}$. The probability of selecting a black marble at random from the same jar is $\frac{1}{4}$. If the jar contains 11 green marbles, find the total number of marbles in the jar.

10. k के किन मानों (किस मान) के लिए निम्न समीकरणों के युग्म का एक अद्वितीय हल है :

x + 2y = 5 और 3x + ky + 15 = 0

Find the value(s) of k so that the pair of equations $x + 2y = 5$ and $3x + ky + 15 = 0$ has a unique solution.

11. दो संपूरक कोणों में से बड़े कोण का मान छोटे कोण के मान से 18° अधिक है । दोनों कोणों के मान ज्ञात कीजिए ।

अथवा

सुमित की आयु उसके बेटे की आयु की तीन गुनी है । पाँच वर्ष के बाद, उसकी आयु अपने बेटे की आयु की ढ़ाई गुना हो जाएगी । इस समय सुमित की आयु कितने वर्ष है ?

The larger of two supplementary angles exceeds the smaller by 18°. Find the angles.

OR

Sumit is 3 times as old as his son. Five years later, he shall be two and a half times as old as his son. How old is Sumit at present ?

12. निम्नलिखित बारंबारता बंटन का बहुलक ज्ञात कीजिए :

वर्ग अंतराल :	25 – 30	30 – 35	35 – 40	40 – 45	45 – 50	50 – 55
बारंबारता :	25	34	50	42	38	14

Find the mode of the following frequency distribution :

Class Interval :	25 – 30	30 – 35	35 – 40	40 – 45	45 – 50	50 – 55
Frequency :	25	34	50	42	38	14

खण्ड स

SECTION C

प्रश्न संख्या 13 से 22 तक प्रत्येक प्रश्न के 3 अंक हैं ।

Question numbers 13 to 22 carry 3 marks each.

13. सिद्ध कीजिए कि $2 + 5\sqrt{3}$ एक अपरिमेय संख्या है, दिया गया है कि $\sqrt{3}$ एक अपरिमेय संख्या है ।

अथवा

यूक्लिड ऐल्गोरिथ्म के प्रयोग से 2048 तथा 960 का म.स. (HCF) ज्ञात कीजिए ।

Prove that $2 + 5\sqrt{3}$ is an irrational number, given that $\sqrt{3}$ is an irrational number.

OR

Using Euclid's Algorithm, find the HCF of 2048 and 960.

कर्ण BC पर एक ही तरफ दो समकोण त्रिभुज ABC तथा DBC बनाए गए हैं । यदि AC तथा BD एक दूसरे को बिंदु P पर प्रतिच्छेद करते हैं, तो सिद्ध कीजिए कि

AP × PC = BP × DP.

### अथवा

एक समलंब PQRS जिसमें PQ ∥ RS है, के विकर्ण परस्पर बिन्दु O पर प्रतिच्छेद करते हैं । यदि PQ = 3RS हो, तो त्रिभुजों POQ तथा ROS के क्षेत्रफलों का अनुपात ज्ञात कीजिए ।

Two right triangles ABC and DBC are drawn on the same hypotenuse BC and on the same side of BC. If AC and BD intersect at P, prove that AP × PC = BP × DP.

OR

Diagonals of a trapezium PQRS intersect each other at the point O, PQ ∥ RS and PQ = 3RS. Find the ratio of the areas of triangles POQ and ROS.

15. आकृति 3 में, PQ तथा RS, O केन्द्र वाले किसी वृत्त पर दो समांतर स्पर्श-रेखाएँ हैं और स्पर्श बिन्दु C पर स्पर्श-रेखा AB, PQ को A तथा RS को B पर प्रतिच्छेद करती है । सिद्ध कीजिए कि ∠ AOB = 90° है ।

आकृति 3
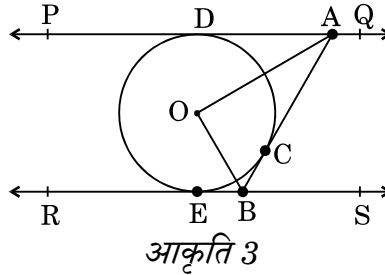
In Figure 3, PQ and RS are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersecting PQ at A and RS at B. Prove that ∠ AOB = 90°.

Figure 3
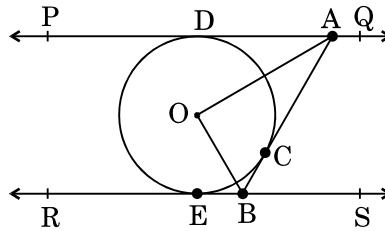
16. रेखा x – 3y = 0 बिन्दुओं (– 2, – 5) तथा (6, 3) को जोड़ने वाले रेखाखंड को किस अनुपात में विभाजित करती है ? इस प्रतिच्छेद बिन्दु के निर्देशांक भी ज्ञात कीजिए ।

Find the ratio in which the line x – 3y = 0 divides the line segment joining the points (– 2, – 5) and (6, 3). Find the coordinates of the point of intersection.
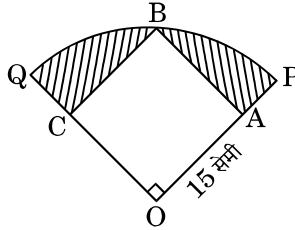
17. मान ज्ञात कीजिए :

$$\left(\frac{3 \sin 43°}{\cos 47°}\right)^2 - \frac{\cos 37° \operatorname{cosec} 53°}{\tan 5° \tan 25° \tan 45° \tan 65° \tan 85°}$$

Evaluate :

$$\left(\frac{3 \sin 43°}{\cos 47°}\right)^2 - \frac{\cos 37° \operatorname{cosec} 53°}{\tan 5° \tan 25° \tan 45° \tan 65° \tan 85°}$$
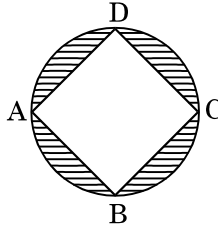
18. आकृति 4 में, एक वृत्त के चतुर्थांश OPBQ के अंतर्गत एक वर्ग OABC बना हुआ है । यदि OA = 15 सेमी है, तो छायांकित क्षेत्र का क्षेत्रफल ज्ञात कीजिए । ($\pi = 3\cdot14$ प्रयोग कीजिए)

आकृति 4

अथवा

आकृति 5 में, $2\sqrt{2}$ सेमी भुजा वाला वर्ग ABCD एक वृत्त के अंतर्गत बना हुआ है । छायांकित क्षेत्र का क्षेत्रफल ज्ञात कीजिए । ($\pi = 3\cdot14$ प्रयोग कीजिए)

आकृति 5

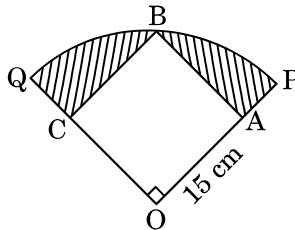In Figure 4, a square OABC is inscribed in a quadrant OPBQ. If OA = 15 cm, find the area of the shaded region. (Use $\pi = 3\cdot14$)

Figure 4

OR

In Figure 5, ABCD is a square with side $2\sqrt{2}$ cm and inscribed in a circle. Find the area of the shaded region. (Use $\pi = 3.14$)
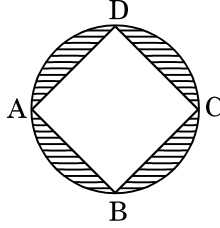

Figure 5

19. एक ठोस बेलन के आकार का है जिसके दोनों सिरे अर्धगोलाकार हैं । ठोस की कुल लम्बाई 20 सेमी है तथा बेलन का व्यास 7 सेमी है । ठोस का कुल आयतन ज्ञात कीजिए । ($\pi = \dfrac{22}{7}$ प्रयोग कीजिए)

A solid is in the form of a cylinder with hemispherical ends. The total height of the solid is 20 cm and the diameter of the cylinder is 7 cm. Find the total volume of the solid. (Use $\pi = \dfrac{22}{7}$)

20. नीचे दिया हुआ बंटन 100 विद्यार्थियों द्वारा एक परीक्षा में प्राप्त अंकों को दर्शा रहा है :

प्राप्तांक :	30 – 35	35 – 40	40 – 45	45 – 50	50 – 55	55 – 60	60 – 65
विद्यार्थियों की संख्या :	14	16	28	23	18	8	3

विद्यार्थियों के माध्य अंक ज्ञात कीजिए ।

The marks obtained by 100 students in an examination are given below :

Marks :	30 – 35	35 – 40	40 – 45	45 – 50	50 – 55	55 – 60	60 – 65
Number of Students :	14	16	28	23	18	8	3

Find the mean marks of the students.

21. k के किस मान के लिए, बहुपद

$$f(x) = 3x^4 - 9x^3 + x^2 + 15x + k,$$

$3x^2 - 5$ से पूर्णतया विभाजित होता है ?

अथवा

द्विघात बहुपद $7y^2 - \dfrac{11}{3}y - \dfrac{2}{3}$ के शून्यक ज्ञात कीजिए और शून्यकों तथा गुणांकों के बीच के संबंध की सत्यता की जाँच कीजिए ।

For what value of k, is the polynomial

$$f(x) = 3x^4 - 9x^3 + x^2 + 15x + k$$

completely divisible by $3x^2 - 5$?

OR

Find the zeroes of the quadratic polynomial $7y^2 - \frac{11}{3}y - \frac{2}{3}$ and verify the relationship between the zeroes and the coefficients.

22. p के ऐसे सभी मान लिखिए कि द्विघात समीकरण $x^2 + px + 16 = 0$ के बराबर मूल हों । प्राप्त समीकरण के मूल ज्ञात कीजिए ।

Write all the values of p for which the quadratic equation $x^2 + px + 16 = 0$ has equal roots. Find the roots of the equation so obtained.

खण्ड द
SECTION D

प्रश्न संख्या 23 से 30 तक प्रत्येक प्रश्न के 4 अंक हैं ।

Question numbers 23 to 30 carry 4 marks each.

23. यदि किसी त्रिभुज की एक भुजा के समांतर अन्य दो भुजाओं को भिन्न-भिन्न बिन्दुओं पर प्रतिच्छेद करने के लिए एक रेखा खींची जाए, तो सिद्ध कीजिए कि ये अन्य दो भुजाएँ एक ही अनुपात में विभाजित हो जाती हैं ।

If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then prove that the other two sides are divided in the same ratio.

24. अमित जो कि एक समतल जमीन पर खड़ा है, अपने से 200 मी. दूर उड़ते हुए पक्षी का उन्नयन कोण 30° पाता है । दीपक जो कि 50 मी. ऊँचे भवन की छत पर खड़ा है, उसी पक्षी का उन्नयन कोण 45° पाता है । अमित और दीपक पक्षी के विपरीत दिशा में हैं । दीपक से पक्षी की दूरी ज्ञात कीजिए ।

Amit, standing on a horizontal plane, finds a bird flying at a distance of 200 m from him at an elevation of 30°. Deepak standing on the roof of a 50 m high building, finds the angle of elevation of the same bird to be 45°. Amit and Deepak are on opposite sides of the bird. Find the distance of the bird from Deepak.

25. लोहे के एक ठोस खंभे में 220 सेमी ऊँचाई के एक बेलन जिसके आधार का व्यास 24 सेमी है, के ऊपर 60 सेमी ऊँचाई का एक अन्य बेलन अध्यारोपित है जिसकी त्रिज्या 8 सेमी है । इस खंभे का भार ज्ञात कीजिए, जबकि दिया गया है कि 1 घन सेमी लोहे का लगभग भार 8 ग्राम है । ($\pi = 3 \cdot 14$ प्रयोग कीजिए)

A solid iron pole consists of a cylinder of height 220 cm and base diameter 24 cm, which is surmounted by another cylinder of height 60 cm and radius 8 cm. Find the mass of the pole, given that 1 cm^3 of iron has approximately 8 gm mass. (Use $\pi = 3 \cdot 14$)

26. 5 सेमी भुजा वाले समबाहु त्रिभुज ABC की रचना कीजिए । फिर एक अन्य त्रिभुज की रचना कीजिए जिसकी भुजाएँ दिए हुए त्रिभुज ABC की संगत भुजाओं की $\frac{2}{3}$ गुनी हों ।

अथवा

2 सेमी त्रिज्या के वृत्त पर 5 सेमी त्रिज्या का एक संकेन्द्री वृत्त खींचिए । बाह्य वृत्त पर लिए गए एक बिन्दु P से छोटे वृत्त पर दो स्पर्श-रेखाओं PA तथा PB की रचना कीजिए । PA की लंबाई मापिए ।

Construct an equilateral Δ ABC with each side 5 cm. Then construct another triangle whose sides are $\frac{2}{3}$ times the corresponding sides of Δ ABC.

OR

Draw two concentric circles of radii 2 cm and 5 cm. Take a point P on the outer circle and construct a pair of tangents PA and PB to the smaller circle. Measure PA.

27. निम्नलिखित बंटन को 'से कम प्रकार' के बंटन में बदलिए और फिर उसका तोरण खींचिए :

वर्ग अंतराल :	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80	80 – 90	90 – 100
बारंबारता :	7	5	8	10	6	6	8

Change the following data into 'less than type' distribution and draw its ogive :

Class Interval :	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80	80 – 90	90 – 100
Frequency :	7	5	8	10	6	6	8

28. सिद्ध कीजिए कि :

$$\frac{\tan \theta}{1 - \cot \theta} + \frac{\cot \theta}{1 - \tan \theta} = 1 + \sec \theta \; \text{cosec} \; \theta$$

अथवा

सिद्ध कीजिए कि :

$$\frac{\sin \theta}{\cot \theta + \text{cosec} \; \theta} = 2 + \frac{\sin \theta}{\cot \theta - \text{cosec} \; \theta}$$

Prove that :

$$\frac{\tan \theta}{1 - \cot \theta} + \frac{\cot \theta}{1 - \tan \theta} = 1 + \sec \theta \; \text{cosec} \; \theta$$

OR

Prove that :

$$\frac{\sin \theta}{\cot \theta + \text{cosec} \; \theta} = 2 + \frac{\sin \theta}{\cot \theta - \text{cosec} \; \theta}$$

29. समांतर श्रेढ़ी –7, –12, –17, –22, ... का कौन-सा पद –82 होगा ? क्या –100 इस समांतर श्रेढ़ी का कोई पद होगा ? सकारण उत्तर बताइए ।

अथवा

समांतर श्रेढ़ी 45, 39, 33, ... के कितने पदों का योगफल 180 होगा ? दोहरे उत्तर की व्याख्या कीजिए ।

Which term of the Arithmetic Progression –7, –12, –17, –22, ... will be –82 ? Is –100 any term of the A.P. ? Give reason for your answer.

OR

How many terms of the Arithmetic Progression 45, 39, 33, ... must be taken so that their sum is 180 ? Explain the double answer.

30. हिन्दी तथा अंग्रेज़ी की कक्षा परीक्षा में अरुण के प्राप्त अंकों का योगफल 30 है । यदि उसके हिन्दी में 2 अंक अधिक होते और अंग्रेज़ी में 3 अंक कम होते, तो प्राप्त अंकों का गुणनफल 210 होता । दोनों विषयों में उसके द्वारा प्राप्त अंकों को ज्ञात कीजिए ।

In a class test, the sum of Arun's marks in Hindi and English is 30. Had he got 2 marks more in Hindi and 3 marks less in English, the product of the marks would have been 210. Find his marks in the two subjects.

तत् त्वं पूषन् अपावृणु
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